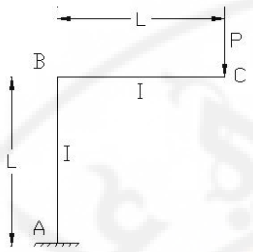
مسائل على نظرية كاستليانو

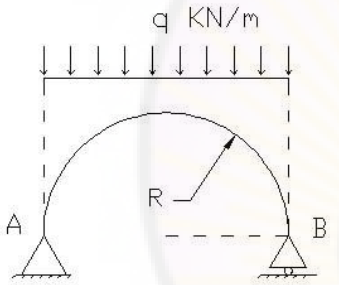
1- أوجد تأثير القوة القاصة Q والقوة الناظمية N على الانتقال الشاقولي للنقطة C.

يعطى : $G = 0.4E$



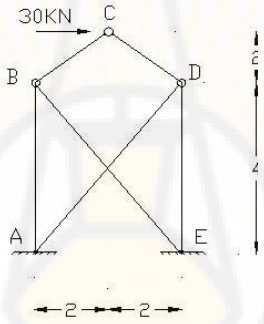
$$\max \frac{h}{L} = \frac{1}{5}$$

2- أوجد الانتقالات المطلوبة في المنشآت التالية :



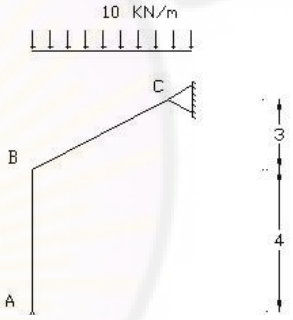
Δ_B

-آ-



Δ_C

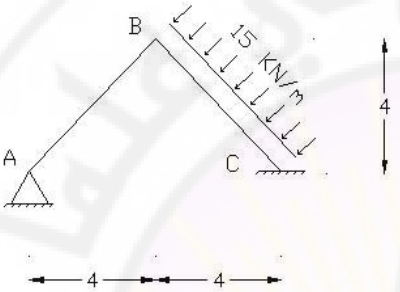
-ب-



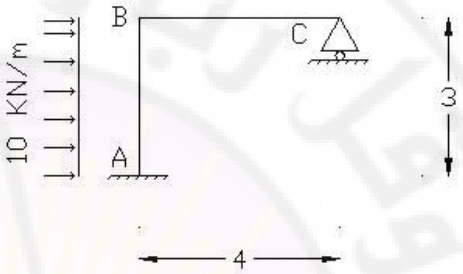
Δ_A

-جـ-

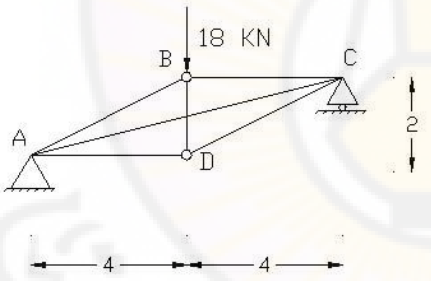
3- يطلب حل المنشآت بطريقة كاستليانو:



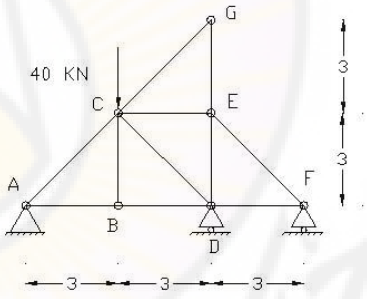
-ب-



-آ-



-د-

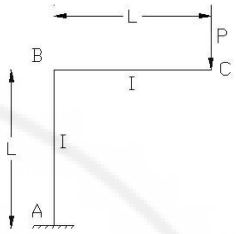


-جـ-

مسألة : (1)

$$U=\int_0^L \frac{1}{2}\frac{(Px)^2}{EI}dx+\int_0^L \frac{1}{2}\frac{(PL)^2}{EI}dy+\int_0^L \frac{1}{2}\frac{P^2}{KAG}dx+\int_0^L \frac{1}{2}\frac{P^2}{EA}dy$$

$$=\frac{PL^3}{3EI}+\frac{PL^3}{EI}+\frac{PL}{KAG}+\frac{PL}{EA}$$



$$=\frac{4}{3}\frac{PL^3}{EI}+\frac{PL}{KAG}+\frac{PL}{AE}$$

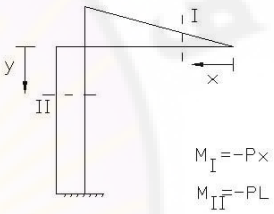
$$=\frac{4}{3}\frac{PL^3}{EI}\left[1+\frac{3}{4}\frac{EI}{KAGL^2}+\frac{3}{4}\frac{I}{AL^2}\right]$$

$$=\frac{4}{3}\frac{PL^3}{EI}\left[1+\frac{3}{4}\frac{bh^3/12}{5/6\,bh(0.4)L^2}+\frac{3}{4}\frac{bh^3/12}{bhL^2}\right]Q$$

$$=\frac{4}{3}\frac{PL^3}{EI}\left[1+\frac{3}{16}\left(\frac{h}{l}\right)^2+\frac{1}{16}\left(\frac{h}{l}\right)^2\right]$$

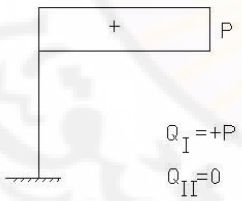
$$=\frac{4}{3}\frac{PL^3}{EI}\left[1+\frac{3}{16}\left(\frac{1}{25}\right)+\frac{1}{16}\left(\frac{1}{25}\right)\right]$$

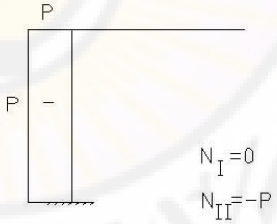
$$=\frac{4}{3}\frac{PL^3}{EI}[\,1+0.0075+0.0025\,]$$



$$1\ \frac{\Delta Q}{\Delta M}\ 4\ 2\ 4\ \frac{\Delta N}{\Delta M}\ 3$$

$$0.01$$





مسألة : (2-آ)

$$M = qR(1-Cos\theta)R + \bar{P}RSin\theta - q\frac{R^2}{2}(1-Cos\theta)^2$$

$$\frac{\partial M}{\partial \bar{P}} = RSin\theta$$

$$\Delta_B = \int \frac{\partial M}{\partial \bar{P}} \frac{M}{EI} dS$$

$$= \int_0^{\pi} RSin\theta \left[(1-Cos\theta) - \frac{1}{2}(1-Cos\theta)^2 \right] qR^2 Rd\theta$$

$$= \frac{qR^4}{EI} \int_0^{\pi} \left[(1-Cos\theta) - \frac{1}{2}(1-Cos\theta)^2 \right] Sin\theta d\theta$$

$$= \frac{qR^4}{EI} \int \left(\mu - \frac{1}{2}\mu^2 \right) d\mu \qquad \mu = 1-Cos\theta \quad d\mu = Sin\theta d\theta$$

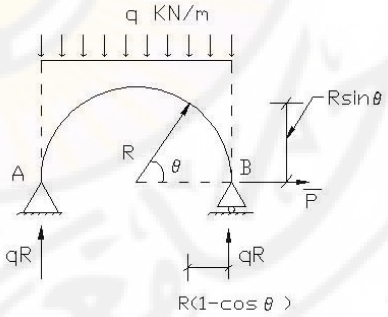
$$= \frac{qR^4}{EI} \left[\frac{\mu^2}{2} - \frac{1}{2}\frac{\mu^3}{3} \right]$$

$$= \frac{qR^4}{EI} \left[\frac{(1-Cos\theta)^2}{2} - \frac{(1-Cos\theta)^3}{6} \right]_0^{\pi}$$

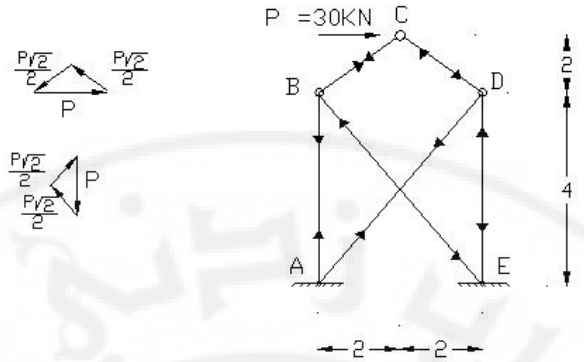
$$= \frac{qR^4}{EI} \left[2 - \frac{8}{6} - \frac{0}{2} + \frac{0}{6} \right]$$

$$= \frac{qR^4}{EI} \frac{12-8}{6}$$

$$= \frac{2qR^4}{3EI}$$



مسألة : (2–ب)



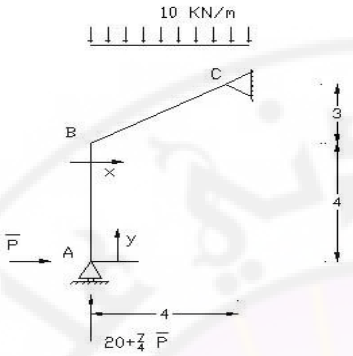
القضيب	N	L	$\frac{\partial N}{\partial P}$
BC	$+P\frac{\sqrt{2}}{2}$	$a\sqrt{2}$	$\frac{\sqrt{2}}{2}$
CD	$-P\frac{\sqrt{2}}{2}$	$a\sqrt{2}$	$-\frac{\sqrt{2}}{2}$
AB	$+P$	$2a$	1
BE	$-P\frac{\sqrt{2}}{2}$	$2a\sqrt{2}$	$-\frac{\sqrt{2}}{2}$
DE	$-P$	$2a$	-1
AD	$+P\frac{\sqrt{2}}{2}$	$2a\sqrt{2}$	$\frac{\sqrt{2}}{2}$

$$\Delta = \frac{1}{EA}\sum \frac{\partial N}{\partial P} NL$$

$$= \frac{1}{EA}\left[\frac{\sqrt{2}}{2}\sqrt{2}\,\frac{\sqrt{2}}{2}(2)+1(2)1(2)+\frac{\sqrt{2}}{2}(2\sqrt{2})\,\frac{\sqrt{2}}{2}(2)\right]Pa$$

$$= \frac{1}{EA}\left[\sqrt{2}+4+2\sqrt{2}\right]Pa = \frac{1}{EA}\left[3\sqrt{2}+4\right]Pa = 8.24\frac{Pa}{EA} = \frac{494.6}{EA}$$

مسألة : (2– جـ)



$$AB: M_I = -\bar{P}y, \frac{\partial M_I}{\partial \bar{P}} = -y, dL = dy$$

$$BC: M_{II} = +\left(20+\frac{7}{4}\bar{P}\right)x - \bar{P}\left(4+\frac{3}{4}x\right) - \frac{10x^2}{2}$$

$$\frac{\partial M_{II}}{\partial \bar{P}} = \frac{7}{4}x - 4 - \frac{3}{4}x = x - 4$$

$$dL = \frac{5}{4}dx$$

$$EI\Delta_A = \int_0^4 0dy + \int_0^4 (20x-5x^2)(x-4)\frac{5}{4}dx$$

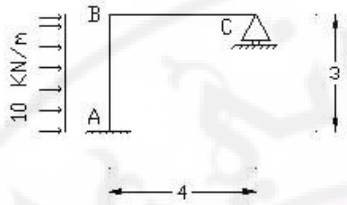
$$= \frac{5}{4}\int_0^4 (20x^2 - 5x^3 - 80x + 20x^2)dx$$

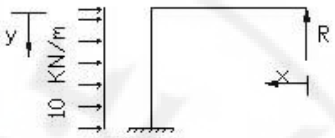
$$= \frac{5}{4}\left[5\frac{x^4}{4} + 40\frac{x^3}{3} - 80\frac{x^2}{2}\right]_0^4$$

$$= \frac{5}{4}4^2\left[-5\frac{4^4}{4}+40\frac{4}{3}-40\right]$$

$$= 20\left[-60+\frac{160}{3}\right] = 20\frac{-180+160}{3} = -\frac{400}{3}$$

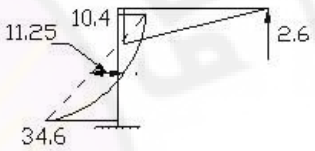
مسالة:(3-آ)





$$BC: M_I = -Rx, \frac{\partial M_I}{\partial R} = x$$

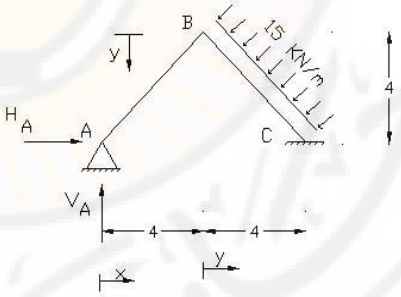
$$AB: M_{II} = 4R - \frac{10y^2}{2}, \frac{\partial M_{II}}{\partial R} = 4$$



$$\frac{\partial U}{\partial R} = 0 = \int_0^4 Rx(x)dx + \int_0^3 (4R - 5y^2)4dy$$

$$R\frac{4^3}{3} + 4^2R(3) - 20\frac{3^3}{3} = 0$$

$$R = \frac{20(27)}{16(13)} = 206$$



مسالة:(3–ب)

$$AB: M_I = V_A x - H_A x$$

$$\frac{\partial M_I}{\partial H_A} = -x, \frac{\partial M_I}{\partial V_A} = x, dL = \sqrt{2}\ dx$$

$$BC: M_{II} = V_A(4+y) - H_A(4-y) - \frac{15(y\sqrt{2})^2}{2}$$

$$\frac{\partial M_{II}}{\partial H_A} = -4+y, \frac{\partial M_{II}}{\partial V_A} = 4+y, dL = \sqrt{2}dy$$

$$(1)\frac{\partial U}{\partial H_A} = 0 = \int_0^4 (V_A x - H_A x)(-x)\sqrt{2}dx + \int_0^4 \left[V_A(4+y) - H_A(4-y) - 15y^2\right](-4+y)\sqrt{2}dy$$

$$(2)\frac{\partial U}{\partial V_A} = 0 = \int_0^4 (V_A x - H_A x)(x)\sqrt{2}dx + \int_0^4 \left[V_A(4+y) - H_A(4-y) - 15y^2\right](4+y)\sqrt{2}dy$$

$$(1)\int_0^4 \left(-V_A x^2 + H_A x^2\right)dx + \int_0^4 \left[V_A\left(y^2-16\right) + H_A\left(16-8y+y^2\right) + 60y^2 - 15y^3\right]dy$$

$$-V_A\frac{4^3}{3}+H_A\frac{4^3}{3}+V_A\frac{4^3}{3}-V_A(16)4+H_A(16)4-H_A(8)\frac{4^3}{3}+H_A\frac{4^3}{3}+60\frac{4^3}{3}-15\frac{4^4}{4}=0$$

$$H_A\frac{4^3}{3}2-V_A(16)4=-60\frac{4^3}{3}+15(4)^3$$

(1) $$2H_A-3V_A=-15$$

$$(2)\int_0^4(V_Ax-H_Ax)x\sqrt{2}dx+\int_0^4\left[V_A(4+y)-H_A(4-y)-15y^3\right](4+y)\sqrt{2}dy=0$$

$$V_A\frac{4^3}{3}-H_A\frac{4^3}{3}+\int_0^4\left[V_A(16+8y+y^2)-H_A(16-y^2)-60y^2-15y^3\right]dy=0$$

$$V_A\frac{4^3}{3}-H_A\frac{4^3}{3}+V_A(16)4+V_A(8)\frac{4^2}{2}+V_A\frac{4^3}{3}-H_A(16)4+H_A\frac{4^3}{3}-60\frac{4^3}{3}-15\frac{4^4}{4}=0$$

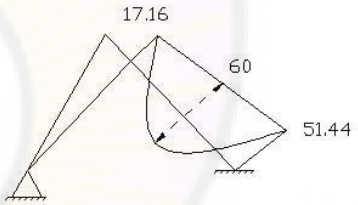
$$-H_A(16)4+V_A4^3\frac{8}{3}=60\frac{4^3}{3}+15(4^3)$$

(2) $$-3H_A+8V_A=105$$

$$\frac{3}{2}\begin{bmatrix}2&-3\\-3&8\end{bmatrix}\begin{bmatrix}H_A\\V_A\end{bmatrix}=\begin{bmatrix}-15\\+105\end{bmatrix}$$

1

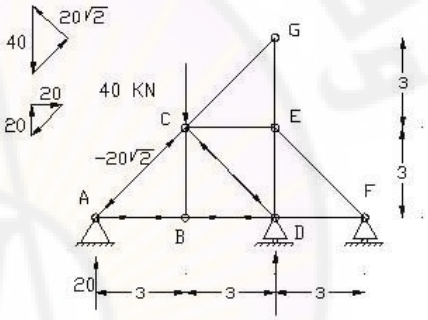
$$3.5V_A=82.5\Rightarrow\begin{matrix}V_A=23.57\\H_A=27.86\end{matrix}$$

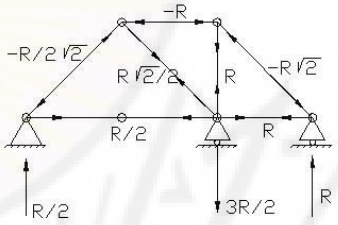


مسألة:(3– جـ)

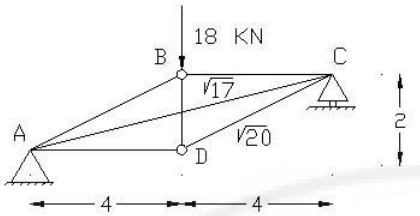
القضيب	N	$\frac{\partial N}{\partial R}$	L	N_{Final}
AC	$-20\sqrt{2}-R\sqrt{2}/2$	$-\sqrt{2}/2$	$3\sqrt{2}$	-26.46
AD	$+20+R/2$	$1/2$	6	+18.71
CE	$-R$	-1	3	+2.583
CD	$-20\sqrt{2}-R\sqrt{2}/2$	$\sqrt{2}/2$	$3\sqrt{2}$	-30.11
DE	$+R$	1	3	-2.583
DF	$+R$	1	3	-2.583
EF	$-R\sqrt{2}$	$-\sqrt{2}$	$3\sqrt{2}$	+3.653

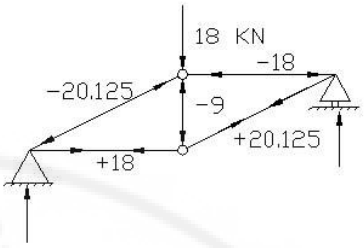
$$\left(-20\sqrt{2}-R\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)3\sqrt{2}$$
$$+\left(-20\sqrt{2}+R\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)3\sqrt{2}$$
$$+\left(20+\frac{R}{2}\right)\left(\frac{1}{2}\right)6$$
$$+(-R)(-1)3(3)$$
$$+\left(-R\sqrt{2}\right)\left(-\sqrt{2}\right)3\sqrt{2}=0$$
$$R\left(+\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+\frac{1}{2}+3+2\sqrt{2}\right)=-20$$
$$R\frac{7+6\sqrt{2}}{2}=-20\Rightarrow R=-2.583$$

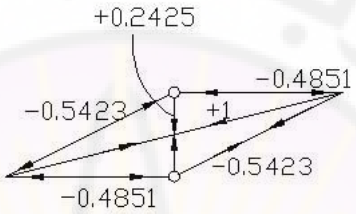




مسالة:(3– د)


18 KN
B
C
A
D
√17
√20
2
4
4


18 KN
-18
-20.125
-9
+20.125
+18


+0.2425
-0.4851
-0.5423
+1
-0.5423
-0.4851

القضيب	N_0	N_1	L	N_{Final}
AB	-20.125	-0.5423	$\sqrt{20}$	-20.309
AC	0	1	$\sqrt{68}$	+0.339
AD	+18	-0.4851	4	+17.836
BC	-18	-0.4851	4	-18.164
BD	-9	+0.2425	2	-8.918
CD	+20.125	-0.5423	$\sqrt{20}$	19.941

الحل وفق طريقة العمل الوهمي :

$$\delta_{10} = \sum N_0 N_1 L = [-9(0.2425)2]$$
$$= -4.365$$

$$\delta_{11} = \left[(0.5423)^2\sqrt{20}(2) + (0.4851)^2 4(2) + 1\sqrt{68} + (0.2425)^2 2\right]$$

$$= 12.8768$$

$$X = 0.339$$

الحل وفق طريقة كاستليانو :

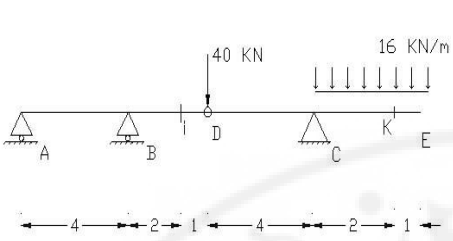
$$\frac{\partial U}{\partial X_1} = 0 = \sum N \frac{\partial N}{\partial X_1} L$$
$$N = N_0 + N_1 X_1$$

$$[\ (-20.125 - 0.5423X_1)(-0.5423)\sqrt{20} + X_1\sqrt{68} + (18 - 0.4854X_1)(-0.4851)4$$
$$+ (+20.125 - 0.5423X_1)(-0.5423)\sqrt{20} + (-18 - 0.4854X_1)(-0.4851)4$$
$$+ (-9 + 0.245X_1)(0.2425)2 = 0$$

نحصل على النتيجة السابقة .

مسائل على العمل الوهمي

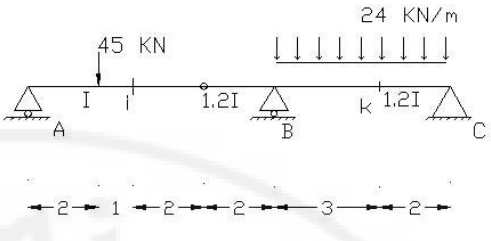
1- أوجد انتقال النقطتين i, k (δ_i, δ_k) في المنشأين المقررين التاليين :



$E = 2.1\times10^5\ KN/cm^2$

$I = 12000\ cm^4$

- آ -

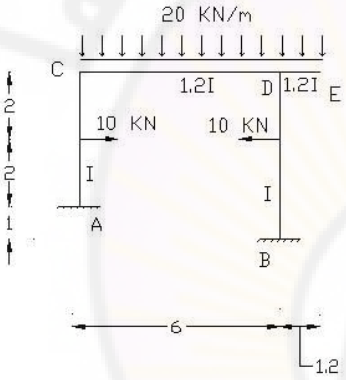


$E = 3000\ KN/cm^2$

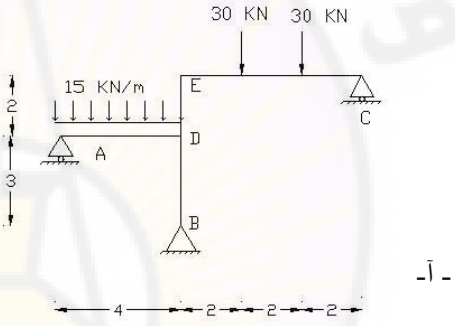
$I = 200000\ cm^4$

- ب -

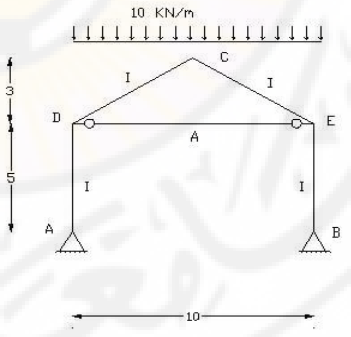
2- حل الإطارت غير المقررة الآتية وارسم مخطط M_{Final} :



- ب -

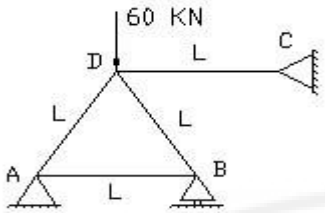


- آ -

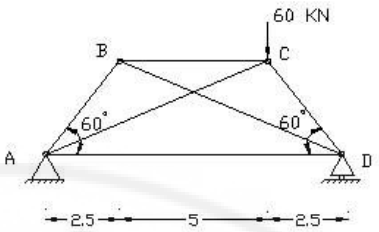


- جـ -

3- حل الجوائز الشبكية الغير مقررة وأوجد N_{Final}

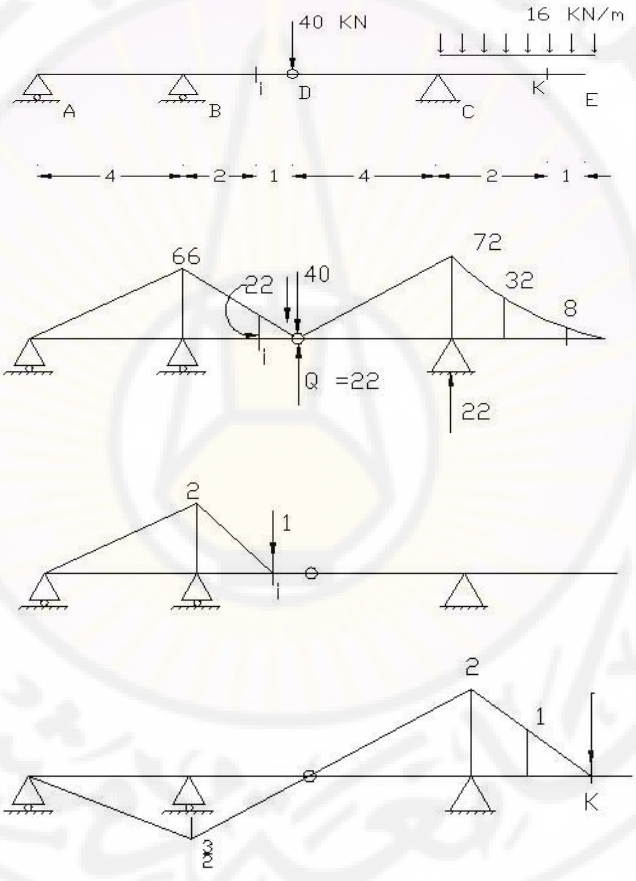


-ب-



-ا-

مسألة (1-آ)



$$EI\delta_i = \frac{1}{2}4(66)\frac{2}{3}2 + \frac{2}{6}[2(66)2 + 22(2)]$$

$$= 176 + \frac{1}{3}(264 + 44)$$

$$\delta_i = \frac{278.66}{2.1\times10^5 \times10^{-3} \times10^6 \times12000\times10^{-8}}$$

$$= 11.06mm$$

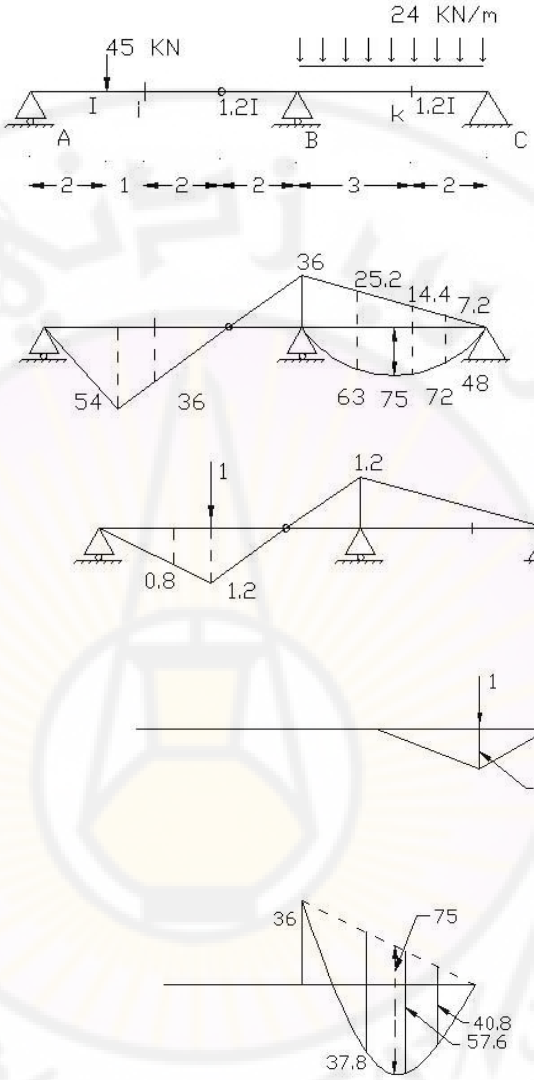
$$EI\delta_k = \frac{1}{3}[72(2) + 4(32)] + \frac{1}{2}4(72)\frac{2}{3}2 - \frac{1}{2}3(66)1 - \frac{1}{2}4(66)1$$

$$= \frac{272}{3} + \frac{576}{3} - 99 - 132$$

$$= \frac{51.667}{2.1\times10^5 \times10^{-3} \times10^6 \times12000\times10^{-8}}$$

$$= 2.05mm$$

مسألة (1–ب)


24 KN/m
45 KN
I
1.2I
k
1.2I
A
B
C
2
1
2
2
3
2
36
25.2
14.4
7.2
54
36
63
75
72
48
1
1.2
0.8
1.2
1
1.2
36
75
40.8
57.6
37.8

$$\delta_i = \frac{1}{EI}\left[\frac{1}{2}2(54)\frac{2}{3}0.8\right] + \frac{1}{EI}\frac{1}{6}\left[2(54)0.8 + 2(36)1.2 + 54(1.2) + 36(0.8)\right]$$
$$+ \frac{1}{EI}\left[\frac{1}{2}2(36)0.8\right] + \frac{1}{1.2EI}\left[\frac{1}{2}2(36)0.8\right] + \frac{1}{1.2EI}\left[\frac{1}{2}5(36)0.8\right] - \frac{1}{1.2EI}\left[\frac{2}{3}5(75)0.6\right]$$

$$= \frac{1}{EI}\left[28.8 + 14.4 + 14.4 + 10.8 + 4.8 + 28.8 + 24 + 60 - 125\right]$$

$$= \frac{61}{3000 \times 10^4 \times 200000 \times 10^{-8}} m$$

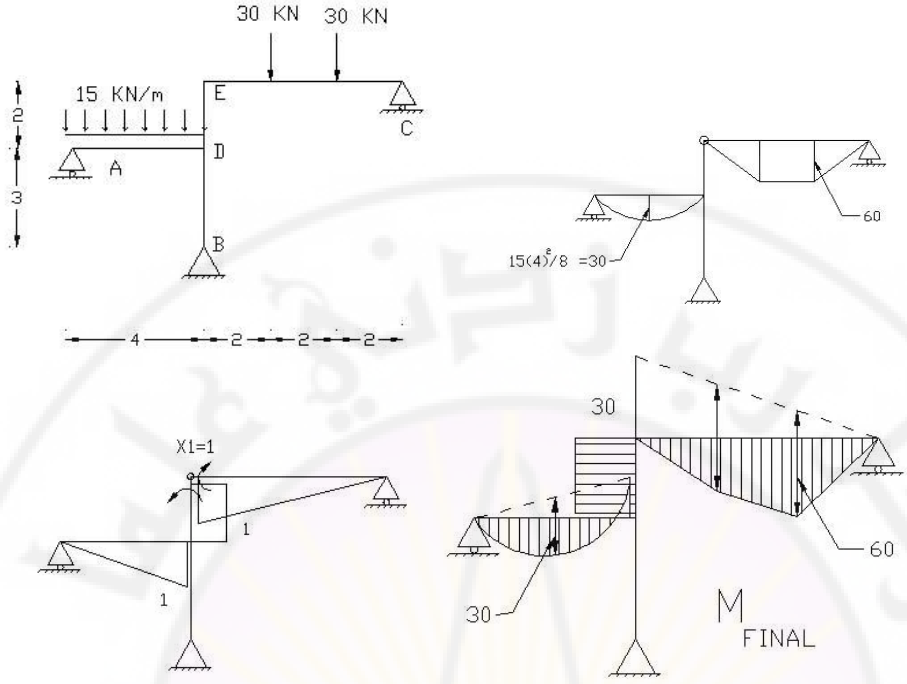
$$= 1.02mm$$

$$1.2EI\delta_k = \frac{1}{3}\left[4(0..6)40.8 + 1.2(57.6)\right] + \frac{1.5}{3}\left[1.2(57.6) + 4(0.6)37.8\right]$$
$$3(1.2)EI\delta_k = 97.92 + 69.12 + 1.5(69.12 + 90.72)$$

$$\delta_k = \frac{406.8}{3(1.2)3000 \times 10^4 \times 200000 \times 10^{-8}} m$$

$$= 1.88mm$$

مسألة (2-آ)



$$EI\delta_{10} = 4(60)\frac{1}{2} + \frac{2}{3}4(30)\frac{1}{2}$$

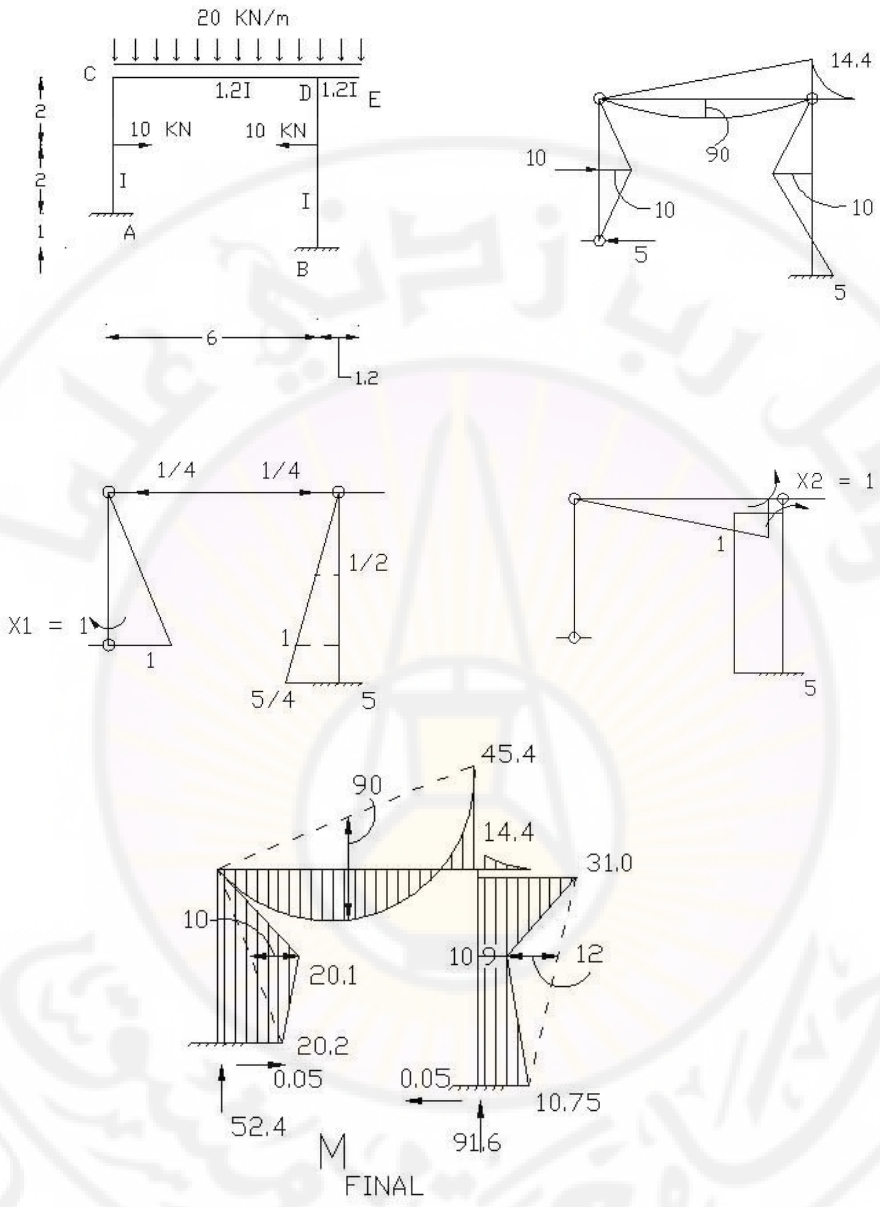
$$= 120 + 40 = 160$$

$$EI\delta_{11} = \frac{1}{2}6\frac{2}{3} + 1(2) + \frac{1}{2}4\frac{2}{3}$$

$$= 4 + \frac{4}{3} = \frac{16}{3}$$

$$x_1 = -\frac{160}{16}3 = 30$$

مسألة (2–ب)


20 KN/m
C
D
E
1.2I
1.2I
10 KN
10 KN
I
I
A
B
2
2
1
6
1.2
14.4
90
10
10
10
5
5
X1 = 1
1/4
1/4
1/2
1
1
5/4
5
X2 = 1
1
5
45.4
90
14.4
31.0
10
20.1
10
9
12
20.2
0.05
0.05
10.75
52.4
91.6
M
FINAL

$$EI\delta_{10} = \frac{1}{2}4(10)\frac{1}{2} + \frac{1}{2}2(10)\frac{2}{3}\frac{1}{2} + \frac{3}{6}\left[2\left(\frac{1}{2}\right)10 - 2(5)\frac{5}{4} + 10\frac{5}{4} - 5\frac{1}{2}\right]$$

$$= 10 + \frac{10}{3} + \frac{1}{2}(10 + 12.5 + 12.5 - 2.5)$$

$$= 17.083$$

$$EI\delta_{20} = \frac{1}{2}4(10) - \frac{1}{2}1(5) + \frac{1}{1.2}\left[\frac{2}{3}6(90)\frac{1}{2}\right] - \frac{1}{1.2}\left[\frac{1}{2}6(14.4)\frac{2}{3}\right]$$

$$= 20 - 2.5 + 150 - 24$$

$$= 143.5$$

$$EI\delta_{11} = \frac{1}{2}4\frac{2}{3} + \frac{1}{2}5\left(\frac{5}{4}\right)\frac{2}{3}\frac{5}{4}$$

$$= \frac{4}{3} + \frac{125}{3(16)}$$

$$= 3.9375$$

$$EI\delta_{12} = \frac{1}{2}5\frac{5}{4}$$

$$= 3.125$$

$$EI\delta_{22} = 5 + \frac{1}{1.2}\left[\frac{1}{2}6\frac{2}{3}\right]$$

$$= 5 + \frac{10}{6}$$

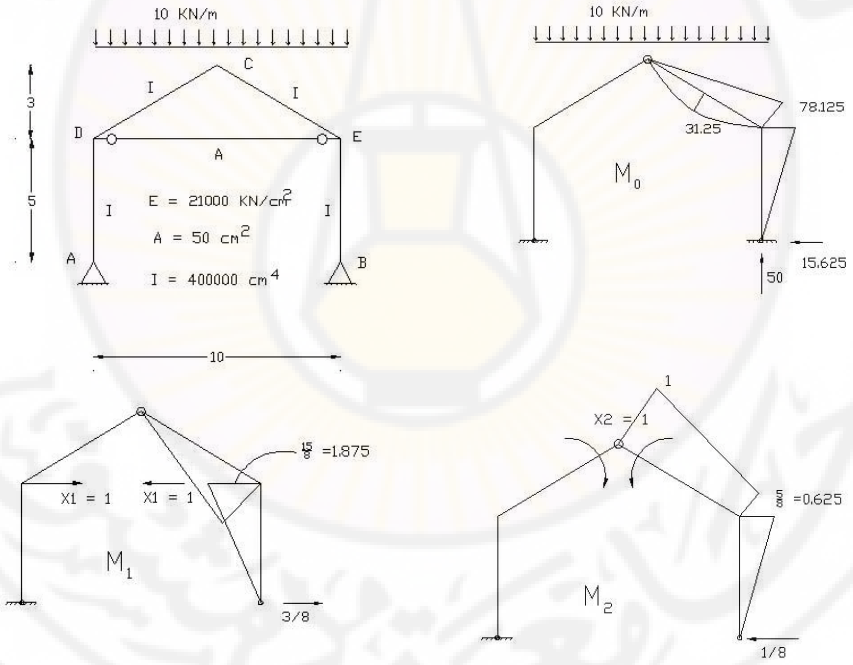
$$= 6.667$$

$$-\frac{3.125}{3.9375}\begin{bmatrix} 3.9375 & 3.125 \\ 3.125 & 6.667 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -17.083 \\ -143.4 \end{bmatrix}$$

$$1$$

$$4.187X_2 = -129.94$$
$$X_2 = -31.0KNm$$
$$X_1 = +20.2KNm$$

مسألة (2-جـ) :



$$EI\delta_{10} = 2\left[-\frac{1}{2}5(78.125)\frac{2}{3}\frac{15}{8} - \frac{1}{2}\sqrt{34}(78.125)\frac{2}{3}\frac{15}{8} + \frac{2}{3}\sqrt{34}(31.25)\frac{1}{2}\frac{15}{8}\right]$$

$$= -829.94$$

$$EI\delta_{20} = 2\left[-\frac{1}{2}5(78.125)\frac{2}{3}\frac{5}{8} + \frac{1}{2}\sqrt{34}(78.125)\left(\frac{1}{3}+\frac{2}{3}\frac{5}{8}\right) - \frac{2}{3}\sqrt{34}(31.25)\frac{1}{2}\left(1+\frac{5}{8}\right)\right]$$

$$= 307.02$$

$$EI\delta_{11} = 2\left[\frac{1}{2}5\frac{15}{8}\frac{2}{3}\frac{15}{8} + \frac{1}{2}\sqrt{34}\frac{15}{8}\frac{2}{3}\frac{15}{8}\right] + \frac{I}{A}(1)(10)$$

$$= 25.385 + \frac{400000}{50}10^{-4}(10)$$

$$= 33.385$$

$$EI\delta_{12} = 2\left[\frac{1}{2}5\frac{15}{8}\frac{2}{3}\frac{15}{8} - \frac{1}{2}\sqrt{34}\frac{15}{8}\left(\frac{1}{3}+\frac{2}{3}\frac{5}{8}\right)\right]$$

$$= -12.106$$

$$EI\delta_{22} = 2\left[\frac{1}{2}5\frac{5}{8}\frac{2}{3}\frac{5}{8} + \frac{\sqrt{34}}{3}\left(1+\frac{5}{8}\frac{5}{8}+\frac{5}{8}\right)\right]$$

$$= 9.1374$$

$$\frac{12.106}{33.385}\begin{bmatrix} 33.385 & -12.106 \\ -12.106 & 9.1374 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 829.94 \\ -307.02 \end{bmatrix}$$

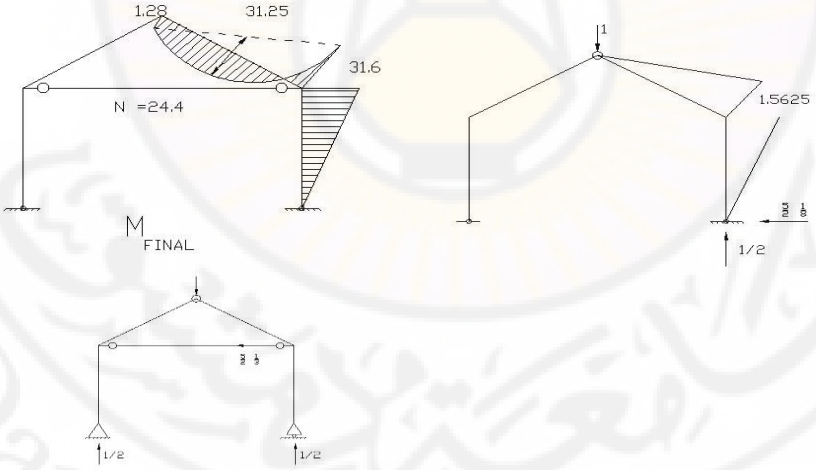
1

$$4.7475X_2 = -6.0689$$
$$X_2 = -1.28KNm$$
$$X_1 = 24.4KN$$

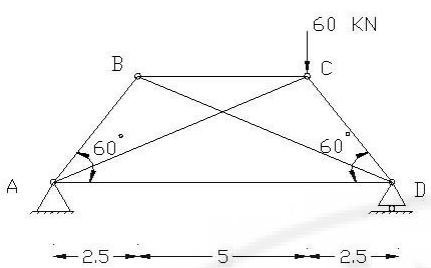
$$EI\delta_C = 2\left[\frac{1}{2}5(31.6)\frac{2}{3}1.5625 + \frac{1}{2}\sqrt{34}(1.5625)\left(\frac{2}{3}31.6 - \frac{1}{3}1.28\right)\right.$$
$$\left. - \frac{2}{3}\sqrt{34}(31.25)\frac{1}{2}1.5625\right]$$
$$= 162.822$$

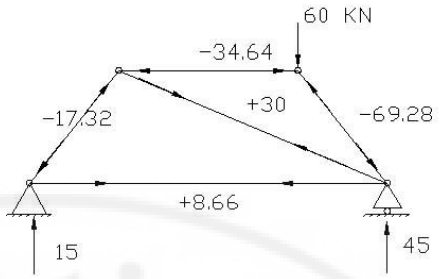
$$\delta_C = \frac{162.822}{21000\times10^4 \times 400000\times10^{-8}}\, m$$
$$= 0.194mm$$

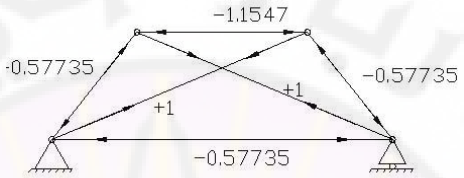
$$\delta_C = \frac{1}{EI}(24.4)\left(\frac{5}{2}\frac{1}{3}\right)(10)m$$
$$= 0.194mm$$



مسألة 3-آ-







القضيب	N_0	N_1	L	N_{Final}
AB	-17.32	-0.57735	5	-4.895
AC	0	+1	8.66	-21.52
AD	+8.66	-0.57735	10	+21.08
BC	-34.64	-1.1547	5	-9.79
BD	30	+1	8.66	+8.48
CD	-69.28	-0.57735	5	-56.86

$$\delta_{10} = [\ -17.32(-0.57735)5 + 8.66(-0.57735)10 - 34.64(-1.1547)5$$
$$+ 30(1)8.66 - 69.28(-0.57735)5\ ]$$

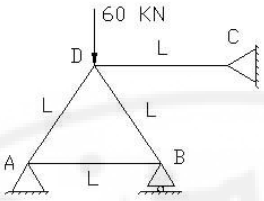
$$= 659.788$$

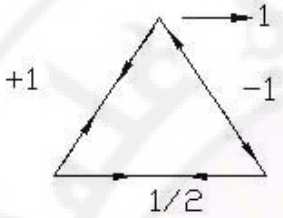
$$\delta_{11} = \left[(0.57735)^2(5+10+5)+8.66(2)+(.15471)^2 5\right]$$

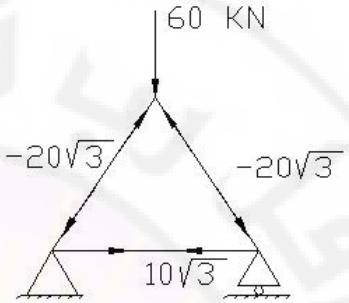
$$= 30.653$$

$$X_1 = -21.52$$

مسألة (3–ب) :







القضيب	N_0	N_1	N_{Final}
AB	$10\sqrt{3}$	+1/2	+15.988
AD	$-20\sqrt{3}$	+1	-37.3057
BD	$-20\sqrt{3}$	-1	-31.9763
CD	0	+1	-2.665

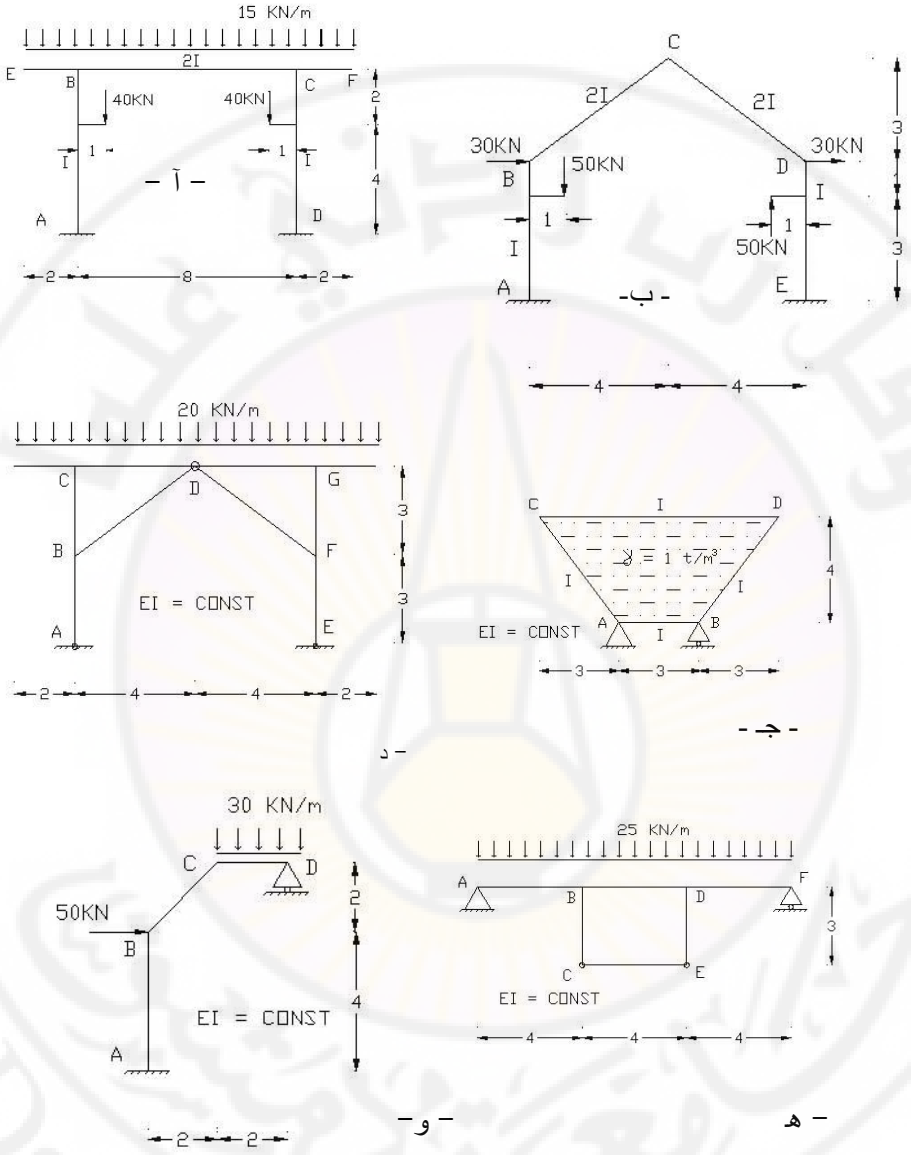
$$\delta_{10} = \sum N_0 N_1 L = \sum N_0 N_1, L = const$$

$$\delta_{11} = \frac{1}{4}+1+1+1 = \frac{13}{4} \qquad = 10\sqrt{3}\frac{1}{2}$$

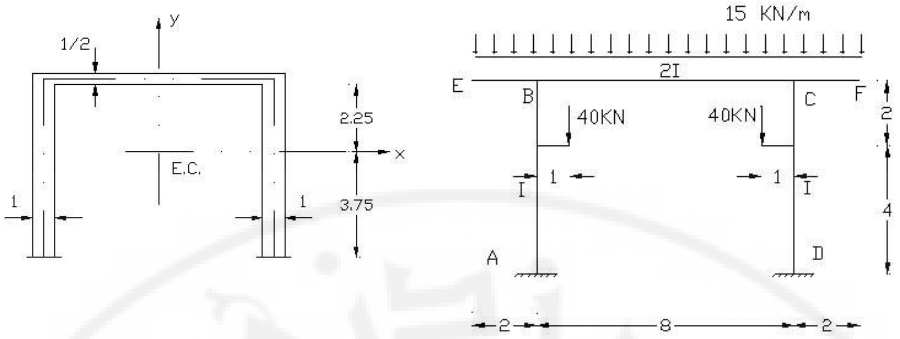
$$X_1 = -10\frac{\sqrt{3}}{2}\frac{4}{13} = -2.665$$

مسائل على المركز المرن والتشبيه العمودي

- يطلب حل المنشآت غير المقررة الآتية بطريقة المركز المرن والتشبيه العمودي



مسألة (آ) :



$$\bar{y} = \frac{6(3)2}{6(2)+\frac{1}{2}8} = \frac{36}{16} = 2.25$$

$$A = 16$$

$$Ix = 2\frac{6^3}{3} - 16(2.25)^2 = 63$$

$$N_1 = -30(8)\frac{1}{2} = -120$$

$$N_2 = -+\frac{2}{3}3(8)(120)\frac{1}{2} = 320$$

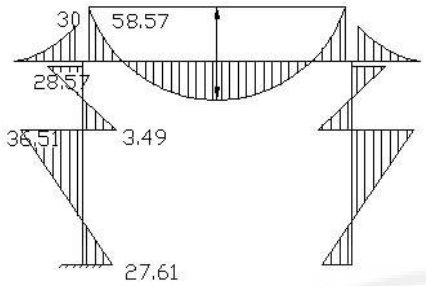
$$N_3 = -40(4) = -160$$

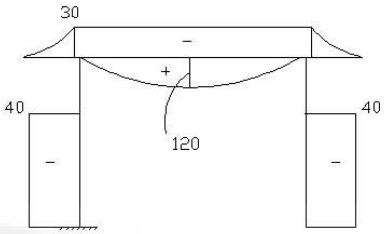
$$N = -120+320-160(2) = -120$$

$$M_x = (-120+320)2.25+160(1.75)2 = -1010$$

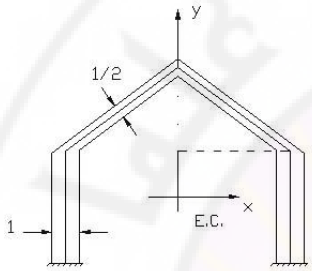
$$M_{Final} = M_0 - \frac{1010}{63}y + \frac{120}{16}$$

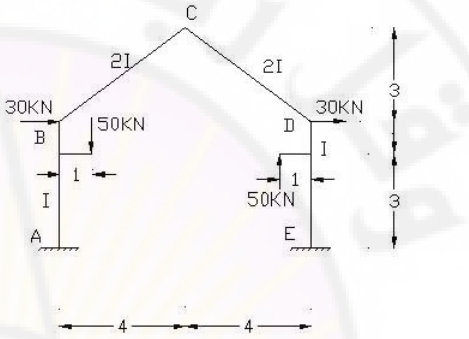
$$= M_0 - 16.032y + 7.5$$





مسألة (ب) :



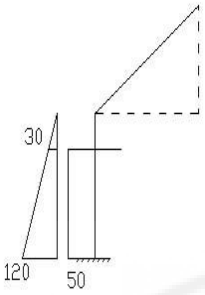


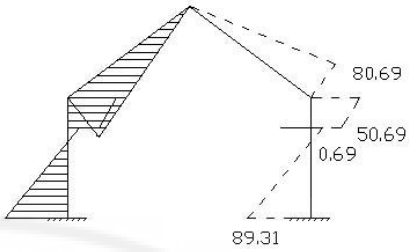
$$I_y = \frac{1}{2}5\frac{4^2}{3}2 + 4(4)^2 2 = 154.67$$

$$M_y = 2\left[3(50) + \frac{1}{2}4(120)\right]4 = 3120$$

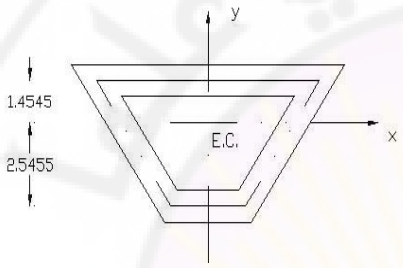
$$M_{Final} = M_0 - \frac{3120}{154.67}x$$

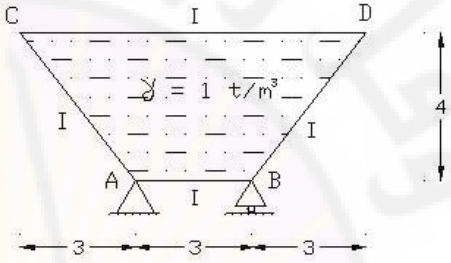
$$= M_0 - 20.17x$$





مسألة (جـ) :





$$\bar{y} = \frac{2(5)2 + 9(4)}{3 - 2(5) + 9} = \frac{56}{22} = 2.5455$$

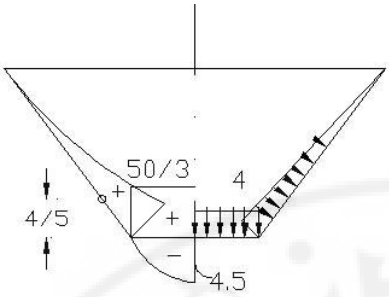
$$I_x = 2(5)\frac{42}{3} + 9(4)^2 - 22(2.5455)^2 = 54.788$$

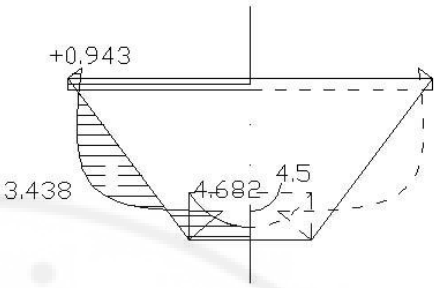
$$N = \frac{1}{4}5\left(\frac{50}{3}\right)2 + \frac{50}{3}(3) - \frac{2}{3}3(4.5) = 41.667 + 50 - 9 = 82.667$$

$$M_x = -41.667(1.7455) - (50 - 9)(2.5455) = -177.09$$

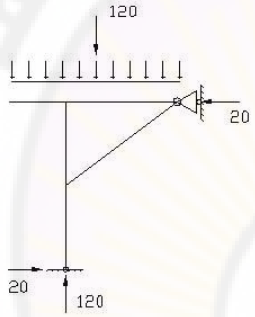
$$M_{Final} = M_0 + \frac{177.09}{54.788}y - \frac{82.667}{22}$$

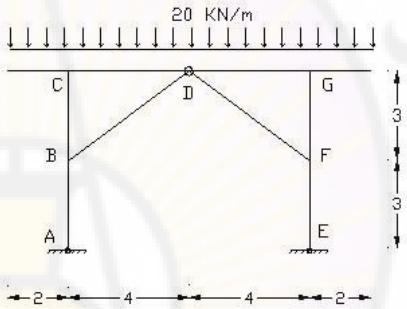
$$= M_0 + 3.232y - 3.758$$





مسألة (د) :





$$I_x = \frac{3^3}{3} + 5\frac{3^3}{3} = 9 + 15 = 24$$

$$I_y = \frac{4^3}{3} + 5\frac{4^3}{3} + 3(4)^2 = 96$$

$$I_{xy} = 3(-1.5)(-4) + 5\frac{3(4)}{12} + 5(-2)(-1.5) = +38$$

$$I'_x = I_x - \frac{I_{xy}{}^2}{I_y} = 8.9583$$

$$I'_y = 35.833$$

$$M_x = \frac{1}{2}3(60)2 = +180$$

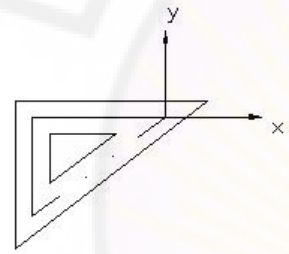
$$M_y = \frac{1}{2}4(40)\frac{2}{3}4 - \frac{2}{3}4(40)2 + \frac{1}{3}3(60)4 = 360$$

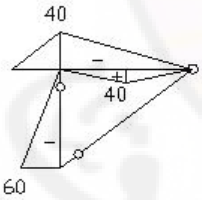
$$M'_x = 180 - 360\frac{38}{96} = 37.5$$

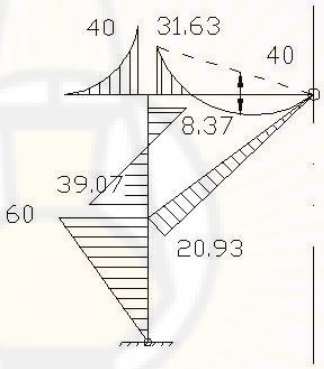
$$M'_y = 360 - 180\frac{38}{24} = 75$$

$$M_{Final} = M_0 - \frac{37.5}{8.9583}y - \frac{75}{35.833}x$$

$$= M_0 - 4.186y - 2.093x$$







مسألة (هـ) :

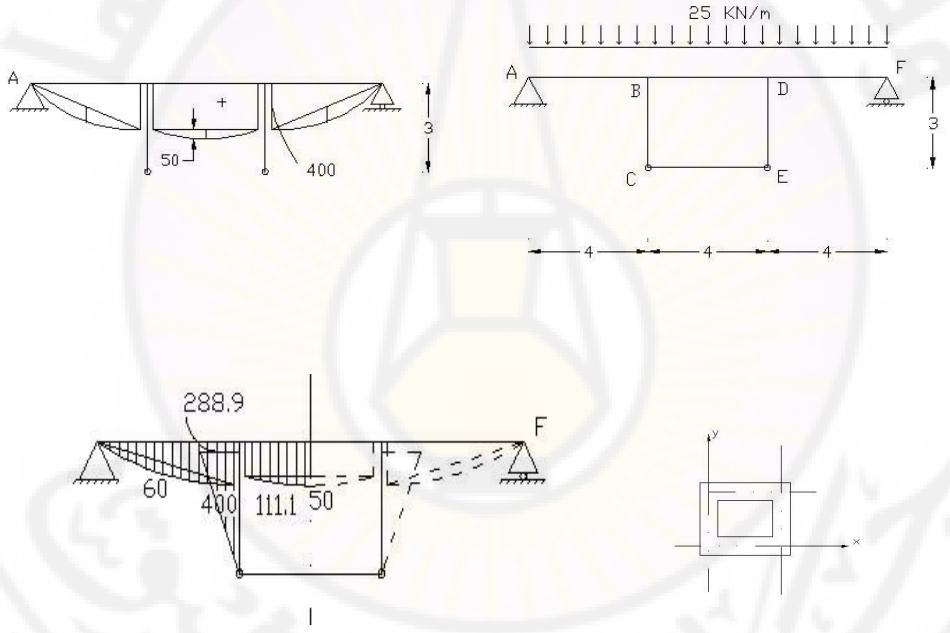
$$I_x = 2\frac{3^3}{3} + 4(3)^2 = 54$$

$$M_x = \left[400(4) + \frac{2}{3}4(50)\right]3 = 5200$$

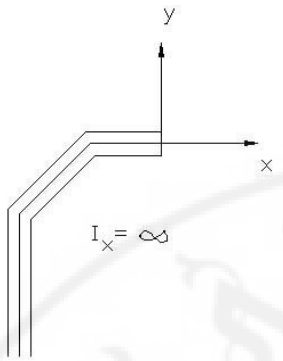
$$M_{Final} = M_0 - \frac{M_x}{I_x}y$$

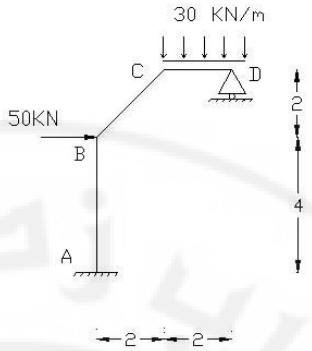
$$M_{Final} = M_0 - \frac{5200}{54}y$$

$$= M_0 - 96.296y$$



مسألة (و) :





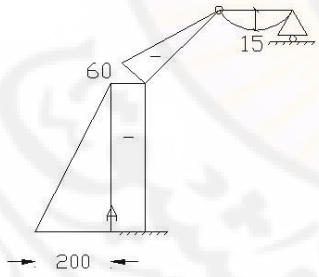
$$I_y = \frac{2^3}{3} + 2\sqrt{2}\frac{2^2}{12} + 2\sqrt{2}(3)^2 + 4(4)^2 = 93.065$$

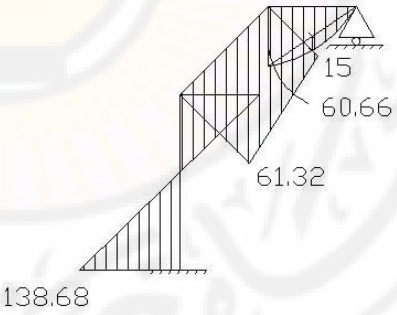
$$M_y = -\frac{2}{3}(2)15(1) + \frac{1}{2}2\sqrt{2}(60)\left(4 - \frac{1}{3}2\right) + \left[60(4) + \frac{1}{2}4(200)\right]4 = 2822.8$$

$$M_{Final} = M_0 - \frac{M_y}{I_y}x$$

$$M_{Final} = M_0 - \frac{2822.8}{93.065}x$$

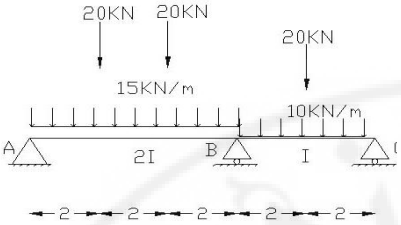
$$= M_0 - 30.33x$$



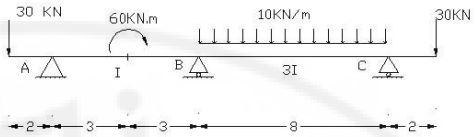


مسائل على طريقة العزوم الثلاثة

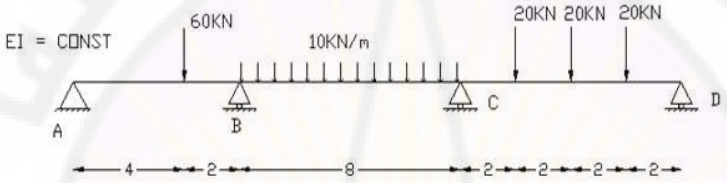
- يطلب حل الجوائز والإطارات الآتية بطريقة العزوم الثلاثة :



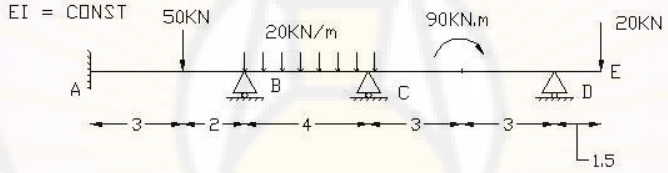
– آ –



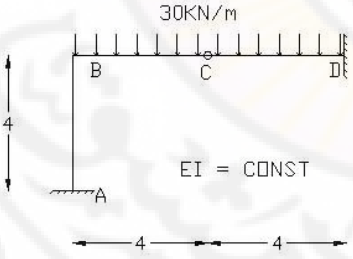
– ب –



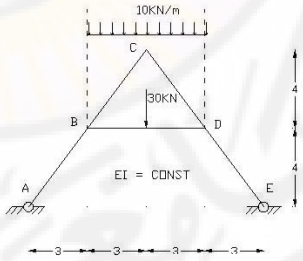
– جـ –



– د –

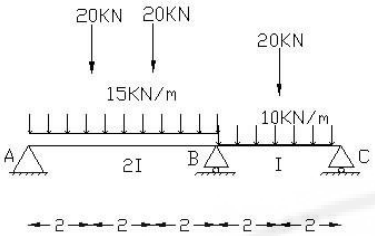


– و –

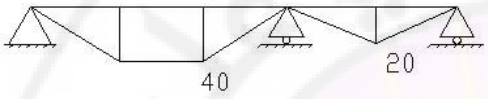


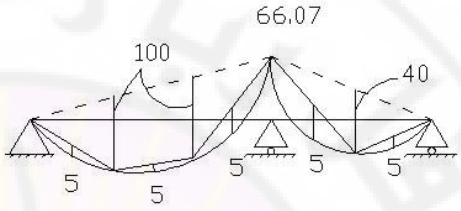
– هـ –

مسألة (آ) :







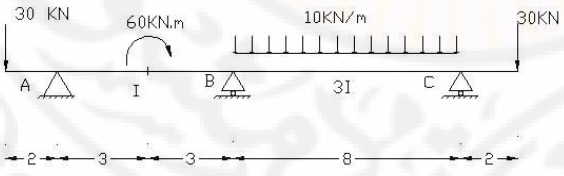


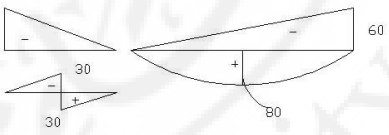
$$2M_B\left(\frac{6}{2}+4\right)=-6\frac{\left[\frac{2}{3}6(67.5)\right]}{2}\frac{1}{2}-6\frac{[4(40)]}{2}\frac{1}{2}-6\frac{\left[\frac{2}{3}4(20)\right]}{1}\frac{1}{2}-6\frac{\left[\frac{1}{2}4(20)\right]}{1}\frac{1}{2}$$

$$14M_B=-405-240-160-120=-925$$

$$M_B=-66.07$$

مسألة (ب) :

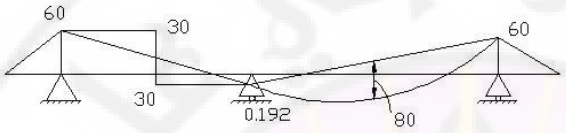




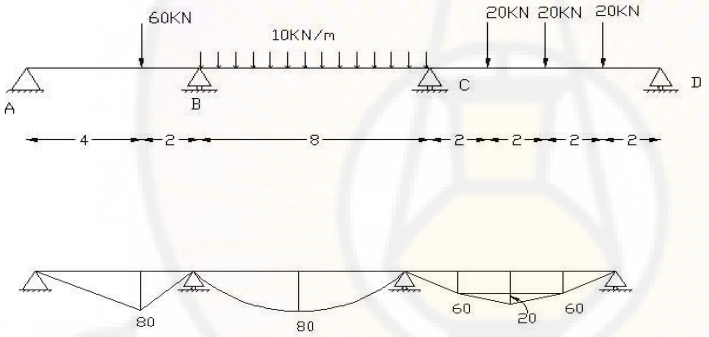
$$2M_B\left(6+\frac{8}{3}\right)=6\frac{\frac{1}{2}6(60)}{1}\frac{1}{3}+6\frac{\frac{1}{2}3(30)}{1}\frac{1}{3}-6\frac{\frac{1}{2}3(30)}{1}\frac{2}{3}-6\frac{\frac{2}{3}8(80)}{3}\frac{1}{2}+6\frac{\frac{1}{2}8(60)}{3}\frac{1}{3}$$

$$52M_B=1080+270-540-1280+480=10$$

$$M_B=+0.192$$



مسألة (جـ) :



$$1/\quad 2M_B(6+8)+M_C(8)=-6\left[\frac{1}{2}6(80)\right]\frac{\frac{2}{3}5}{6}-6\left[\frac{2}{3}8(80)\right]\frac{1}{2}$$

$$(1)\quad 28M_B+8M_C=-800-1280=-2080$$

$$2/\quad M_B(8)+2M_C(8+8)=-6\left[\frac{2}{3}8(80)\right]\frac{1}{2}-6[6(60)+2(20)]\frac{1}{2}$$

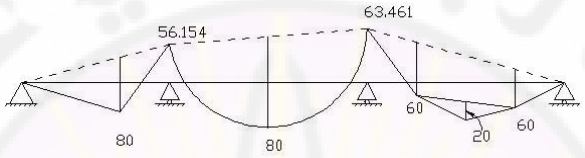
$$(2)\quad 8M_B+32M_C=-1280-1200=-2480$$

$$\begin{matrix}4\\-1\end{matrix}\begin{bmatrix}28 & 8\\ 8 & 32\end{bmatrix}\begin{bmatrix}M_B\\ M_C\end{bmatrix}=\begin{bmatrix}-2080\\ -2480\end{bmatrix}$$

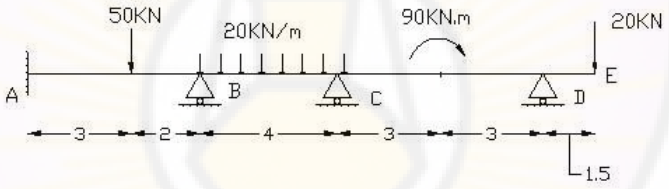
$$104M_C=-5840$$

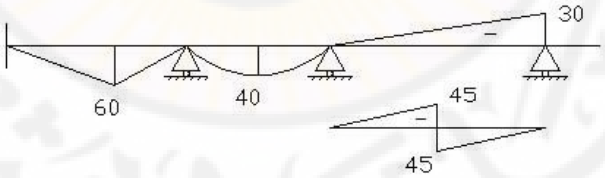
$$M_B=-56.154$$

$$M_C=-63.461$$



مسألة (د) :





$$1/ \quad 2M_A(5)+M_B(5)=-6\left[\frac{1}{2}5(60)\right]\frac{\frac{2}{3}3.5}{5}$$

$$(1) \quad 10M_A+5M_B=-420$$

$$2/ \quad M_A(5)+2M_B(5+4)+M_C(4)=-6\left[\frac{1}{2}5(60)\right]\frac{\frac{2}{3}4}{5}-6\left[\frac{2}{3}4(40)\right]\frac{1}{2}$$

$$(2) \quad 5M_A+18M_B+4M_C=-480-320=-800$$

$$3/ \quad M_B(4)+2M_C(4+6)=-6\left[\frac{2}{3}4(40)\right]\frac{1}{2}+6\left[\frac{1}{2}6(30)\right]\frac{1}{2}+6\left[\frac{1}{2}3(45)\right]\left(\frac{2}{3}-\frac{1}{3}\right)$$

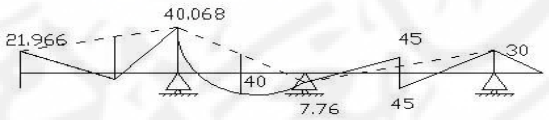
$$(3) \quad 4M_B+20M_C=-320+180+135=-5$$

$$-\frac{5}{10}\begin{bmatrix}10 & 5 & 0\\ 5 & 18 & 4\\ 0 & 4 & 20\end{bmatrix}\begin{bmatrix}M_A\\ M_B\\ M_C\end{bmatrix}=\begin{bmatrix}-420\\ -800\\ -5\end{bmatrix}$$

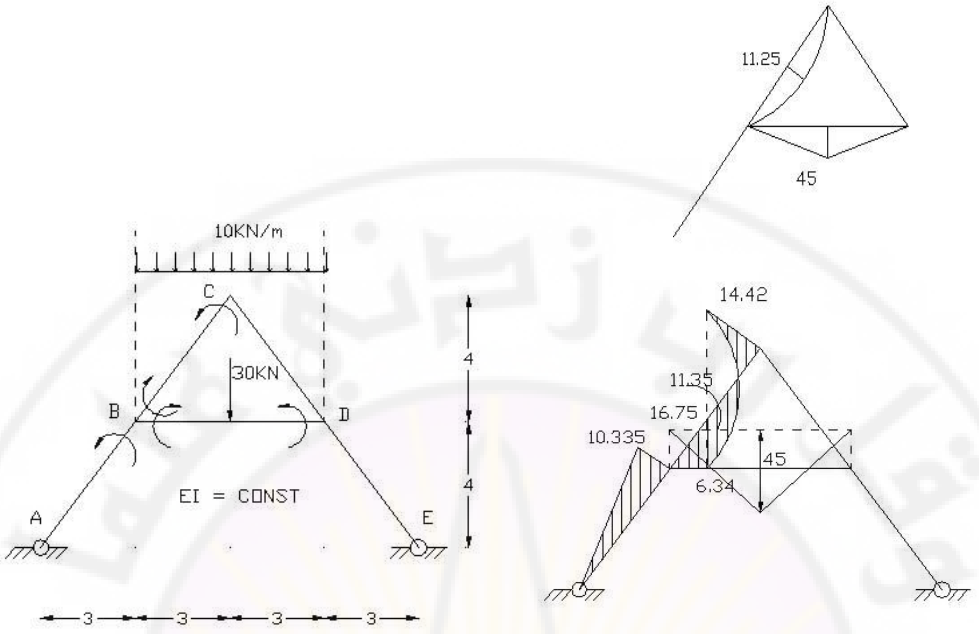
$$-\frac{4}{15.5}\begin{bmatrix}10 & 5 & 0\\ 0 & 15.5 & 4\\ 0 & 4 & 20\end{bmatrix}\begin{bmatrix}M_A\\ M_B\\ M_C\end{bmatrix}=\begin{bmatrix}-420\\ -590\\ -5\end{bmatrix}$$

$$\begin{bmatrix}10 & 5 & 0\\ 0 & 15.5 & 4\\ 0 & 0 & 18.968\end{bmatrix}\begin{bmatrix}M_A\\ M_B\\ M_C\end{bmatrix}=\begin{bmatrix}-420\\ -590\\ 147.26\end{bmatrix}$$

$$M_C=7.763$$
$$M_B=-40.068$$
$$M_A=-21.966$$



مسألة (هـ) :



$$1/\quad 2M_{BA}(5)+2M_{BD}(6)+M_{BD}=-6\left[\frac{1}{2}6(45)\right]\frac{1}{2}$$

$$(1)\quad 10M_{BA}+18M_{BD}=-405$$

$$2/\quad 2M_{BA}(5)+2M_{BC}(5)+M_{C}(5)=-6\left[\frac{2}{3}5(11.25)\right]\frac{1}{2}$$

$$(2)\quad 10M_{BA}+10M_{BC}+5M_{C}=-112.5$$

$$(3)\quad M_{BA}-M_{BD}-M_{BC}=0$$

$$4/\quad M_{BC}(5)+2M_{C}(5)=-6\left[\frac{2}{3}5(11.25)\right]\frac{1}{2}$$

$$(4)\quad M_{BC}+2M_{C}=-22.5$$

$$\begin{matrix} & & \\ -1 & -1 \\ \frac{1}{-} & \frac{1}{-} \\ & \end{matrix}\begin{bmatrix} 1 & 1.8 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} M_{BA} \\ M_{BD} \\ M_{BC} \\ M_C \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -40.5 \\ 0 \\ -11.25 \\ -22.5 \end{bmatrix} \end{bmatrix}$$

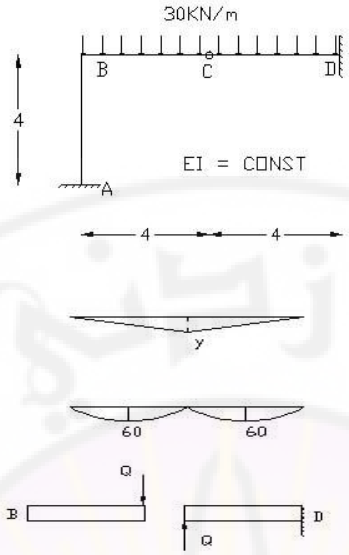
$$\begin{matrix} \\ -\frac{1.8}{2.8} \\ 1 \\ \text{—} \end{matrix}\begin{bmatrix} 1 & 1.8 & 0 & 0 \\ 0 & -2.8 & -1 & 0 \\ 0 & -1.8 & 1 & 0.5 \\ 0 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} M_{BA} \\ M_{BD} \\ M_{BC} \\ M_C \end{bmatrix} = \begin{bmatrix} -40.5 \\ +40.5 \\ +29.25 \\ -22.5 \end{bmatrix}$$

$$\begin{matrix} \\ -\frac{1}{1.643} \\ 1 \\ \text{—} \end{matrix}\begin{bmatrix} 1 & 1.8 & 0 & 0 \\ 0 & -2.8 & -1 & 0 \\ 0 & 0 & 1.643 & 0.5 \\ 0 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} M_{BA} \\ M_{BD} \\ M_{BC} \\ M_C \end{bmatrix} = \begin{bmatrix} -40.5 \\ +40.5 \\ +3.214 \\ -22.5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1.8 & 0 & 0 \\ 0 & -2.8 & -1 & 0 \\ 0 & 0 & 1.643 & 0.5 \\ 0 & 0 & 0 & 1.696 \end{bmatrix}\begin{bmatrix} M_{BA} \\ M_{BD} \\ M_{BC} \\ M_C \end{bmatrix} = \begin{bmatrix} -40.5 \\ +40.5 \\ +3.214 \\ -24.46 \end{bmatrix}$$

$$M_C = -14.42$$
$$M_{BC} = +6.34$$
$$M_{BD} = -16.73$$
$$M_{BA} = -10.385$$

مسألة (و) :



1/ $2M_A(4)+M_B(4)=0$

(1) $2M_A+M_B=0$

2/ $M_A(4)+2M_B(4+4)=-6\left[\frac{2}{3}4(60)\right]\frac{1}{2}-6EI\frac{y}{4}$

(2) $M_A+4M_B=-120-\frac{3}{8}EIy$

3/ $2M_D(4)=-6\left[\frac{2}{3}4(60)\right]\frac{1}{2}-6EI\frac{y}{4}$

(3) $2M_D=-120-\frac{3}{8}EIy$

$(2)-(3)\Rightarrow M_A+4M_B-2M_D=0$

$$M_B = -30(4)2 - Q(4)$$

$$M_D = -30(4)2 + Q(4)$$

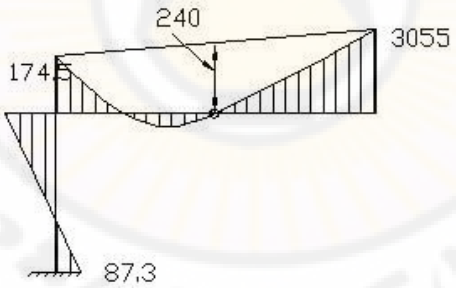
(4) $M_B + M_D = -480$

$$-\frac{1}{2}\begin{bmatrix}2 & 1 & \\ 1 & 4 & -2 \\ & 1 & 1\end{bmatrix}\begin{bmatrix}M_A \\ M_B \\ M_D\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ -480\end{bmatrix}$$

$$-\frac{1}{3.5}\begin{bmatrix}2 & 1 & 0 \\ 0 & 3.5 & -2 \\ & 1 & 1\end{bmatrix}\begin{bmatrix}M_A \\ M_B \\ M_D\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ -480\end{bmatrix}$$

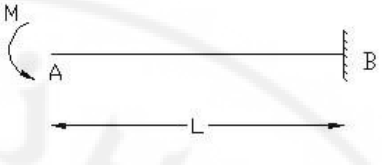
$$\begin{bmatrix}2 & 1 & 0 \\ 0 & 3.5 & -2 \\ 0 & 0 & 1.571\end{bmatrix}\begin{bmatrix}M_A \\ M_B \\ M_D\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ -480\end{bmatrix}$$

$$M_D = -305.5$$
$$M_B = +174.5$$
$$M_A = -87.3$$



مسائل على نظرية كاستليانو

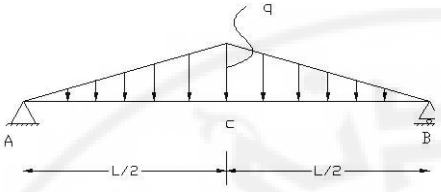
1- استخدم نظرية كاستليانو لحساب الانتقالات المطلوبة في المنشآت الآتية :



انتقال A الشاقولي

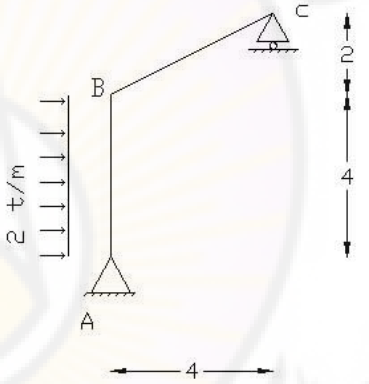
دوران A

- آ -



انتقال C الشاقولي

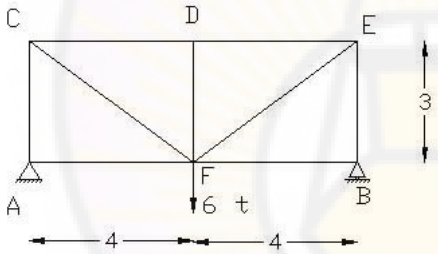
- ب -



انتقال C الشاقولي

دوران B

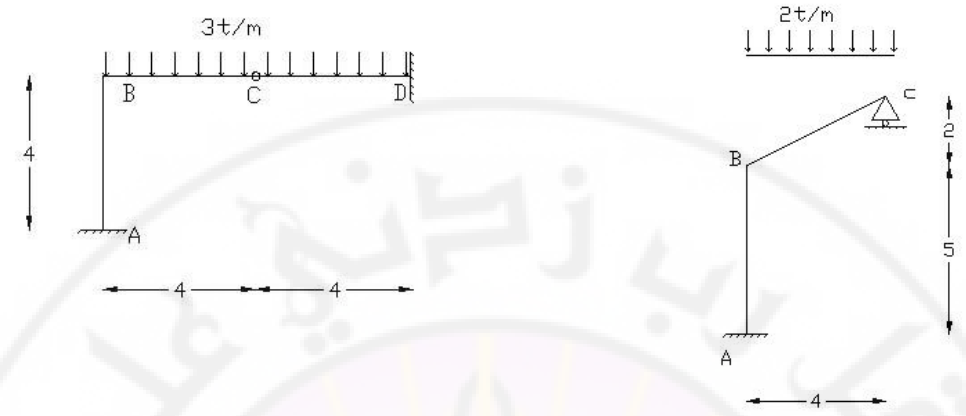
- جـ -



انتقال F الشاقولي

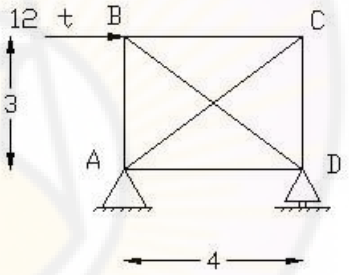
- د -

2- استخدم نظرية كاستليانو لحل المنشآت غير المقررة الآتية :

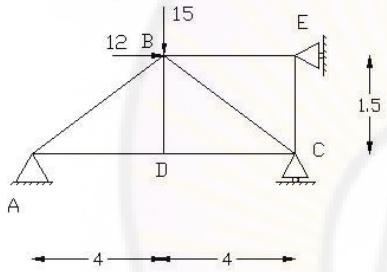


- آ -

- ب -

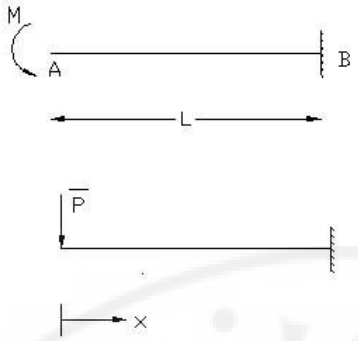


- جـ -



- د -

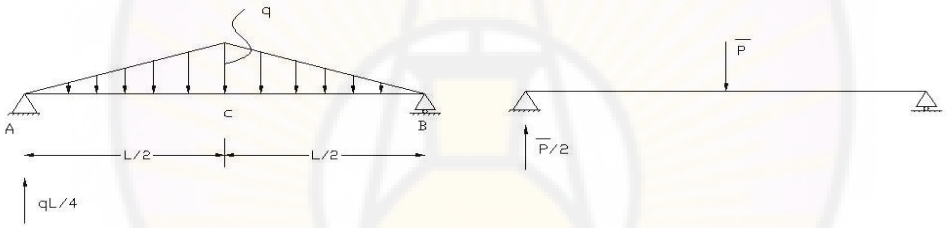
مسألة (1 – آ) :



$$\underline{\theta_A}: M_x = -M, \frac{\partial M_x}{\partial M} = -1, \theta_A = \frac{1}{EI}\int_0^L -M(-1)dx = \frac{ML}{EI}$$

$$\underline{\Delta_A}: M_x = -M - \bar{P}x, \frac{\partial M_x}{\partial \bar{P}} = -x, \Delta_A = \frac{1}{EI}\int_0^L -M(-x)dx = \frac{ML^2}{2EI}$$

مسألة (1 – ب) :



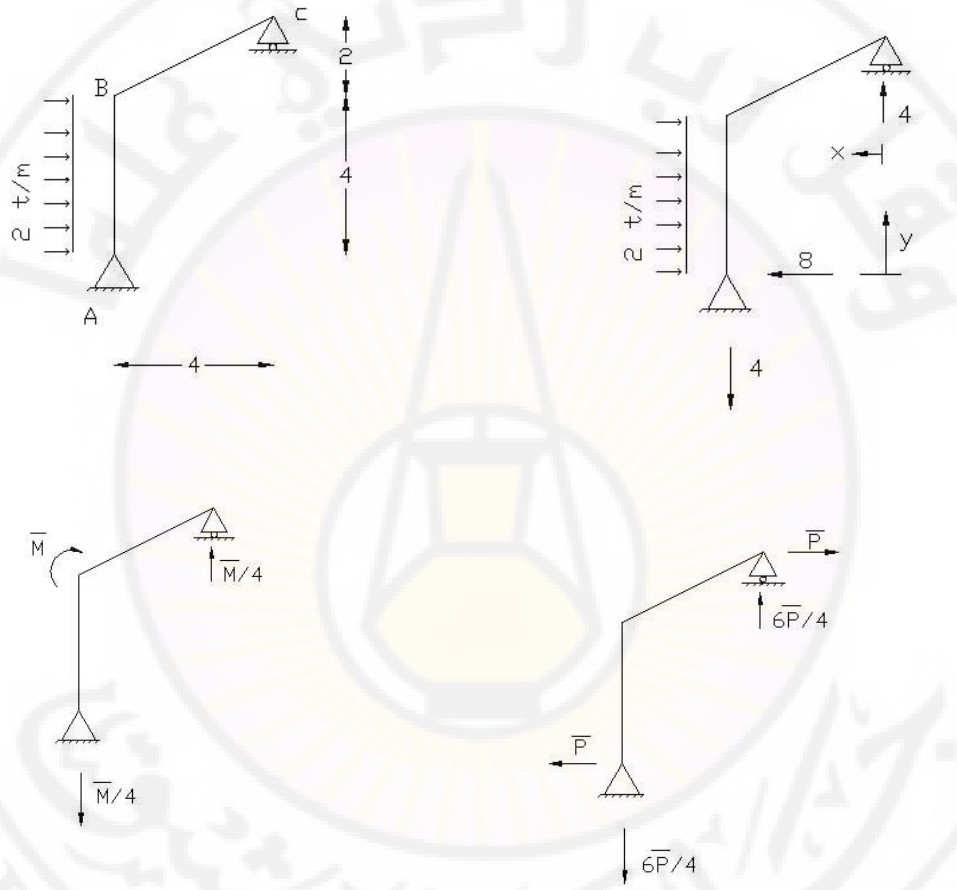
$$M_x = \left(\frac{qL}{4} + \frac{\bar{P}}{2}\right)x - \frac{2q}{L}x\frac{x}{2}\frac{x}{3} = \left(\frac{qL}{4} + \frac{\bar{P}}{2}\right)x - \frac{qx^3}{3L}, \frac{\partial M_x}{\partial \bar{P}} = \frac{x}{2}$$

$$\Delta = \frac{2}{EI}\int_0^{L/2}\left(\frac{qL}{4}x - \frac{qx^3}{3L}\right)\left(\frac{x}{2}\right)dx = \frac{1}{EI}\int_0^{L/2}\left(\frac{qL}{4}x^2 - q\frac{x^4}{3L}\right)dx$$

$$= \frac{1}{EI}\left[\frac{qL}{4}\frac{x^3}{3} - \frac{qx^5}{15L}\right]_0^{L/2} = \frac{qL^4}{EI}\left[\frac{1}{12}\frac{1}{8} - \frac{1}{15}\frac{1}{32}\right]$$

$$= \frac{4}{480}\frac{qL^4}{EI} = \frac{qL^4}{120EI}$$

مسألة (1 – جـ) :



$$\underline{\theta_B}: \quad \underline{AB}: M_y = 8y - \frac{2y^2}{2}, \frac{\partial M_y}{\partial \overline{M}} = 0$$

$$\underline{BC}: M_x = \left(4 + \frac{\overline{M}}{4}\right)x, \frac{\partial M_x}{\partial \overline{M}} = \frac{x}{4}$$

$$EI\theta_B = 0 + \int_0^4 (4x)\left(\frac{x}{4}\right)\frac{\sqrt{20}}{4}\,dx = \frac{\sqrt{20}}{4}\int_0^4 x^2 dx$$

$$= \frac{\sqrt{20}}{4}\frac{4^3}{3} = \frac{\sqrt{20}(16)}{3} = 23.85$$

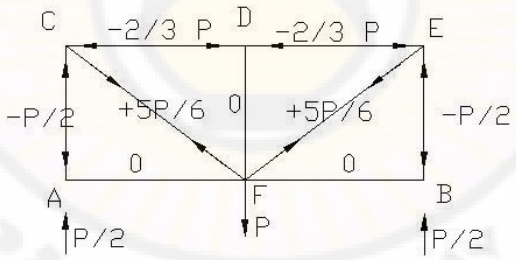
$$\underline{\Delta_C}: \quad \underline{AB}: M_y = (8 + \overline{P})y - y^2, \frac{\partial M_y}{\partial \overline{P}} = y$$

$$\underline{BC}: M_x = \left(4 + \frac{6\overline{P}}{4}\right)x - \overline{P}\frac{2}{4}x = 4x + \overline{P}x, \frac{\partial M_x}{\partial \overline{P}} = x$$

$$EI\Delta_C = \int_0^4 (8y - y^2)y\,dy + \int_0^4 (4x)(+x)\frac{\sqrt{20}}{4}\,dx$$

$$= \frac{8(4)^3}{3} - \frac{4^4}{4} + \sqrt{20}\frac{4^3}{3} = \frac{4^3(8 - 3 + \sqrt{20})}{3} = 202.1$$

مسألة (1 – د) :



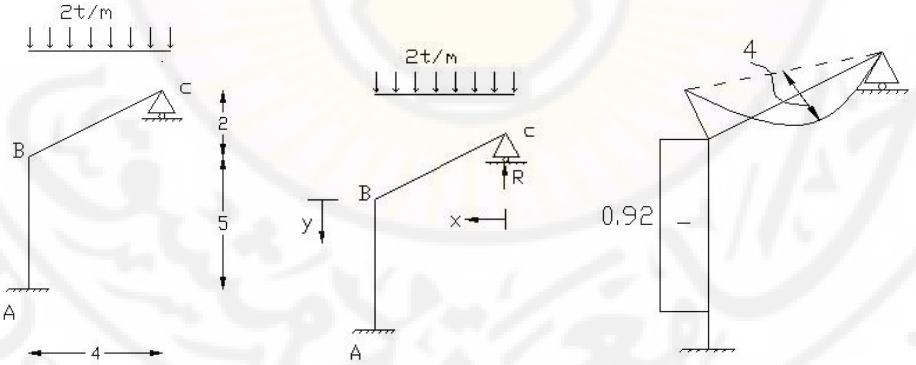
القضيب	N	$\frac{\partial N}{\partial P}$	L
AC	$-P/2$	-1/2	3
AF	0	0	4
BE	$-P/2$	-1/2	3
BF	0	0	4
CD	$-2P/3$	-2/3	4
CF	$+5P/6$	+5/6	5
DE	$-2P/3$	-2/3	4
DF	0	0	3
EF	$+5P/6$	+5/6	5

$$EA\Delta_F = \left[\left(-\frac{P}{2}\right)\left(-\frac{1}{2}\right)3(2)+\left(-\frac{2P}{3}\right)\left(-\frac{2}{3}\right)4(2)+\left(\frac{5P}{6}\right)\left(\frac{5}{6}\right)5(2)\right]$$

$$= P\left[\frac{3}{2}+\frac{32}{9}+\frac{125}{18}\right]$$

$$= \frac{27+64+125}{18}P = 12P = 72$$

مسألة (2 – آ) :



$$\underline{BC}: M_x = Rx - \frac{2x^2}{2}, \frac{\partial M_x}{\partial R} = x, dL = \frac{\sqrt{20}}{4}dx$$

$$\underline{AB}: M_y = 4R - 16, \frac{\partial M_y}{\partial R} = 4$$

$$\frac{\partial U}{\partial R} = \sum \int \frac{M}{EI}\frac{\partial M}{\partial R}dL = 0$$

$$\int_0^4 (Rx - x^2)x\frac{\sqrt{20}}{4}dx + \int_0^5 (4R - 16)4dy = 0$$

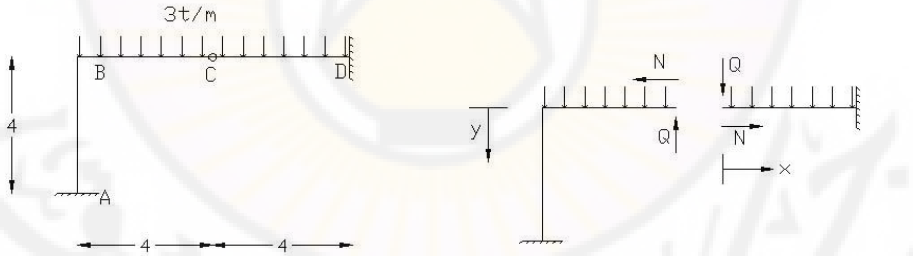
$$\frac{\sqrt{20}}{4}\left[R\frac{4^3}{3} - \frac{4^4}{4}\right] + 4[4R5 - 16(5)] = 0$$

$$\left(\frac{\sqrt{20}(4)^2}{3} + 80\right)R = \sqrt{20}(4)^2 + 320$$

$$R = 3.77$$

$$M_B = -0.9187$$

مسألة (2 – ب) :



$$\underline{BD}: M_x = Q_x - \frac{3x^2}{2}, \frac{\partial M_x}{\partial Q} = -x \frac{\partial M_x}{\partial N} = 0$$

$$\underline{AB}: M_y = N_y + 4Q - 24, \frac{\partial M_y}{\partial Q} = 4, \frac{\partial M_x}{\partial N} = y$$

$$\frac{\partial U}{\partial Q} = 0, \int_{-4}^{+4} \left(-Qx - \frac{3}{2}x^2 \right)(-x)dx + \int_0^4 (Ny + 4Q - 24)4dy = 0$$

$$\int_{-4}^{+4} \left(Qx^2 + \frac{3}{2}x^3 \right) dx + \left[4N\frac{4^2}{2} + 4^2 Q4 - 4(24)4 \right] = 0$$

$$2Q\frac{4^3}{3} + 4^3 Q + N\frac{4^3}{2} - 6(4)^3 = 0$$

(1) $$\frac{5}{3}Q + \frac{1}{2}N = 6$$

$$\frac{\partial U}{\partial N} = 0, \int_{-4}^{+4} dx + \int_0^4 (Ny + 4Q - 24)ydy = 0$$

$$N\frac{4^3}{3} + 4Q\frac{42}{2} - 24\frac{4^2}{2} = 0$$

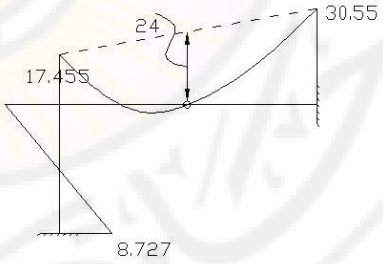
(2) $$\frac{1}{2}Q + \frac{1}{3}N = 3$$

$$-\frac{3}{10}\begin{bmatrix} 10 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} Q \\ N \end{bmatrix} = \begin{bmatrix} 36 \\ 18 \end{bmatrix}$$

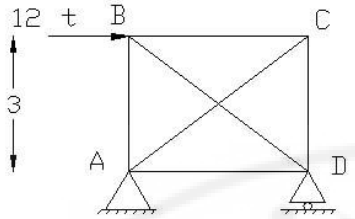
$$1.1N = 7.2$$

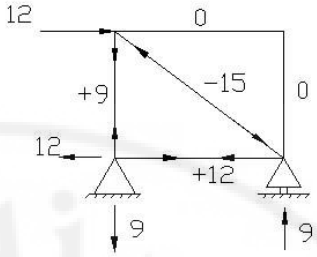
$$N = 6.545$$

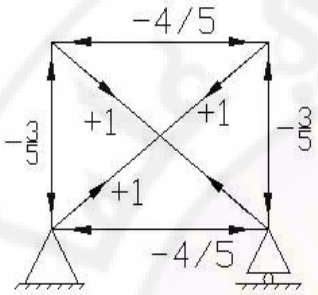
$$Q = 1.636$$



مسألة (2 – جـ) :







$$N = N_0 + X_1 N_1$$

القضيب	N_0	$N_1[X_1]$	L	$\frac{\partial N}{\partial X_1}$	N
AB	+9	-3/5	3	-3/5	+4.5
AC	0	+1	5	1	+7.5
AD	+12	-4/5	4	-4/5	+6
BC	0	-4/5	4	-4/5	-6
BD	-15	+1	5	+1	-7.5
CD	0	-3/5	3	-3/5	-4.5

$$\frac{\partial U}{\partial X_1}=0,\sum\frac{1}{EA}N\frac{\partial N}{\partial X_1}L$$

$$\sum N\frac{\partial N}{\partial X_1}L=0$$

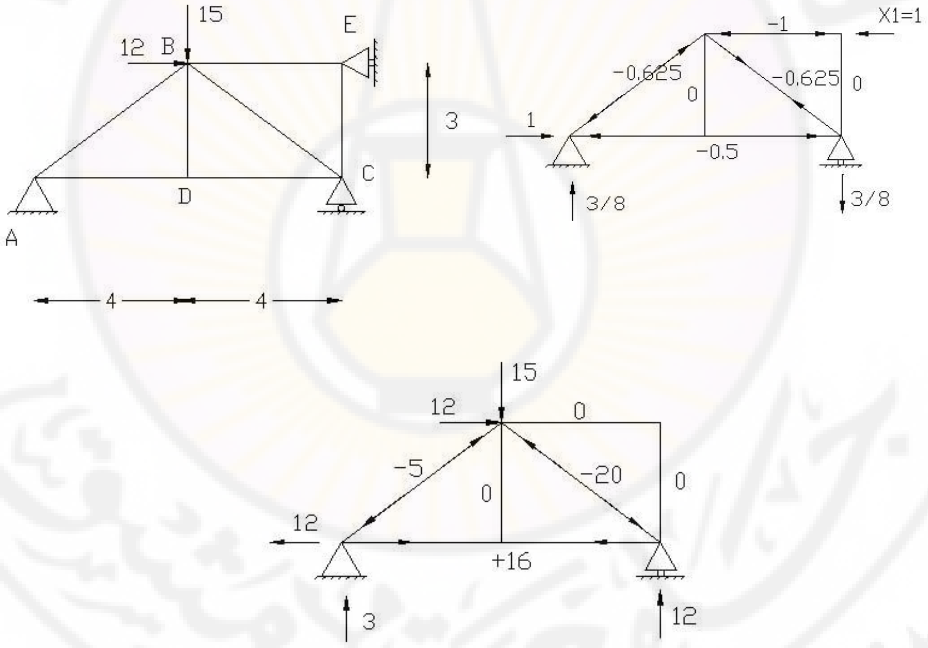
$$\left[\left(9-\frac{3}{5}X_1\right)\left(-\frac{3}{5}\right)3+\left(12-\frac{4}{3}X_1\right)\left(-\frac{4}{5}\right)4+\left(-15+X_1\right)(1)5+X_1 5+X_1\left(\frac{4}{5}\right)^2 4+X_1\left(\frac{3}{5}\right)^2 3\right]=0$$

$$X_1\left[\left(\frac{3}{5}\right)^2 3+\left(\frac{4}{5}\right)^2 4+5+5+\left(\frac{4}{5}\right)^2 4+\left(\frac{3}{5}\right)^2 3\right]$$

$$=9\frac{3}{5}3+12\frac{4}{5}4+15(5)$$

$$X_1=\frac{129.6}{17.28}=7.5$$

مسألة (2 – د) :



$$N = N_0 + X_1 N_1$$

القضيب	N_0	$N_1[X_1]$	L	$\frac{\partial N}{\partial X_1}$	N
AB	-5	-0.625	5	-0.625	-11.995
AD	+16	-0.5	4	-0.5	+10.404
BC	-20	+0.625	5	+0.625	-13.005
BD	0	0	3	0	0
BE	0	-1	4	-1	-11.192
CD	+16	-0.5	4	0.5	+10.404
CE	0	0	3	0	0

$$\frac{\partial U}{\partial X_1} = 0, \sum \frac{N}{EA} \frac{\partial N}{\partial X_1} L$$

$$\sum N \frac{\partial N}{\partial X_1} L = 0$$

$$[(-5 - 0.625X_1)(-0.625)5 + (16 - 0.5X_1)(-0.5)4$$

$$(-20 + 0.625X_1)(0.625)5 + 0 + 4X_1 + (16 - 0.5X_1)(-0.5)4 + 0] = 0$$

$$X_1[(0.625)^2 5(2) + (0.5)^2 4(2) + 4]$$

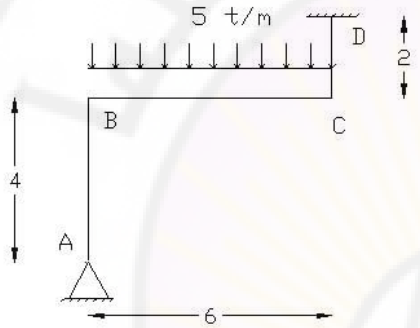
$$= [-5(0.625)5 + 16(0.5)4(2) + 20(0.625)5]$$

$$X_1 9.90625 = 110.875$$

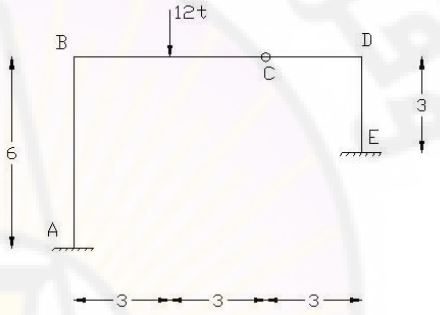
$$X_1 = 11.192$$

مسائل على نظرية العمل الوهمي

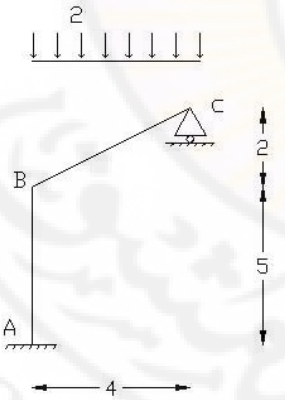
- يطلب حل المنشآت غير المقررة باستخدام طريقة العمل الوهمي:

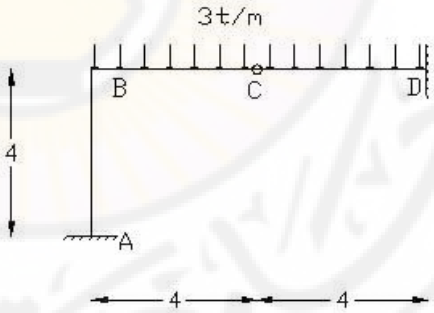


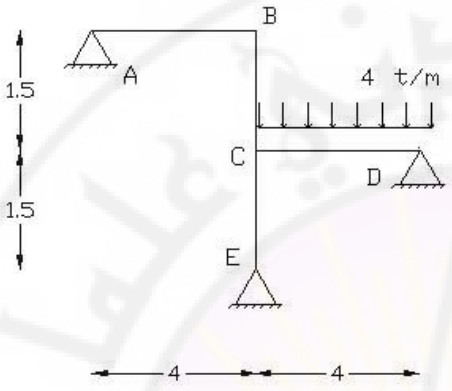
- ب -



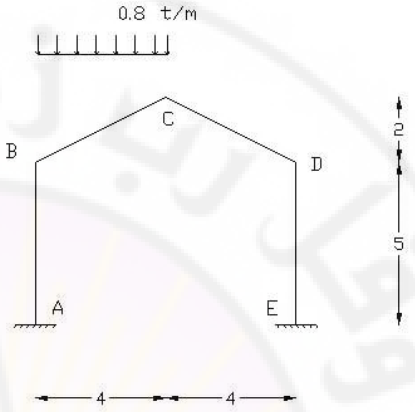
- جـ -





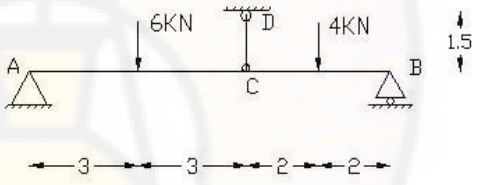


– د –

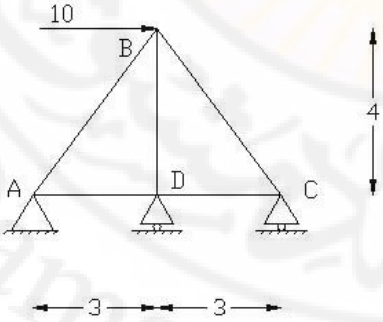


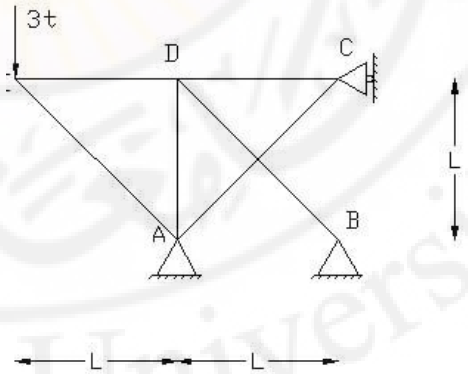
– و –

الجائز من الخشب مقطعه 150mm
عرض 200 mm ارتفاع
$E_{timber}=10^7 KN/m^2$
القضيب CD من الفولاذ : مساحته
150 mm^2
$E_{steel}=2x10^8 KN/m^2$



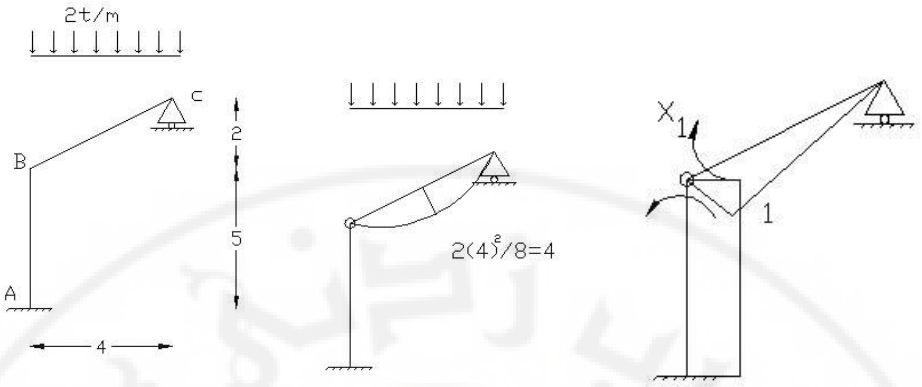
– ز –





Damascus University

مسألة (آ) :



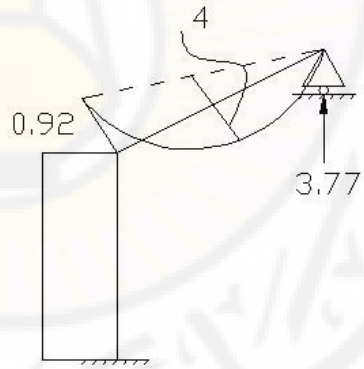
$$\delta_{10} = \frac{2}{3}4(4.472)\frac{1}{2}$$

$$= 5.928$$

$$\delta_{11} = \frac{1}{2}(4.472)\frac{2}{3} + 5$$

$$= 6.4907$$

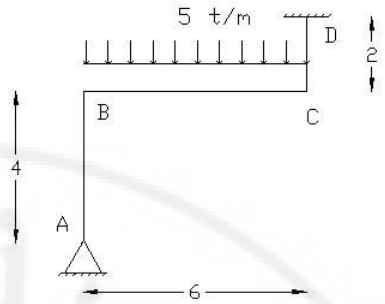
$$X_1 = -0.9187$$



مسألة (ب) :

$$\delta_{10} = \frac{2}{3}6(22.5)\frac{1}{2} = 45$$

$$\delta_{20} = 45$$



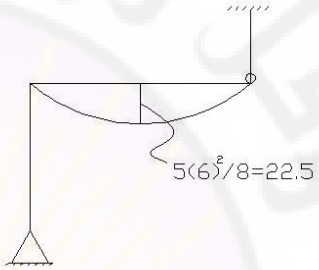
$$\delta_{11} = \frac{1}{2}4\left(\frac{2}{3}\right)+\frac{1}{2}4\left(\frac{2}{3}\right)+\frac{1}{2}2\frac{1}{2}\left(\frac{2}{3}\frac{1}{2}\right)$$

$$= 2+\frac{4}{3}+\frac{1}{6} = \frac{21}{6} = \frac{7}{2} = 3.5$$

$$\delta_{12} = \frac{1}{2}6\left(\frac{1}{3}\right)+\frac{1}{2}2\frac{1}{2} = 1.5$$

$$\delta_{22} = \frac{1}{2}6\frac{2}{3}+1(2) = 4$$

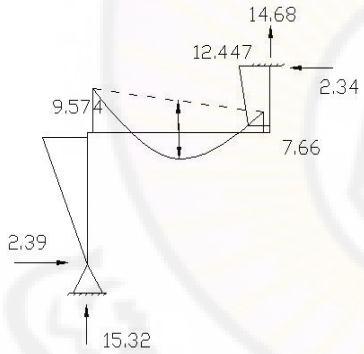
$$\begin{bmatrix} 3.5 & 1.5 \\ 1.5 & 4 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -45 \\ -45 \end{bmatrix}$$

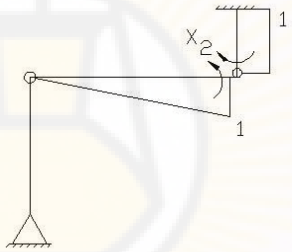


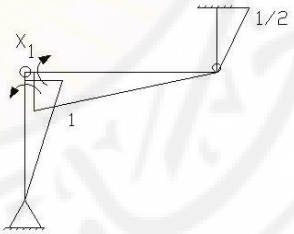
$$3.35714X_2 = -25.7143$$

$$X_1 = -7.660$$

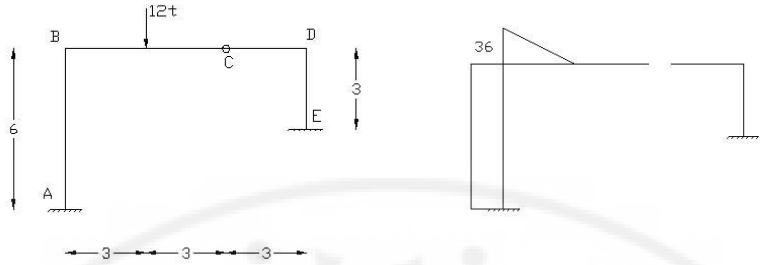
$$X_2 = -9.574$$



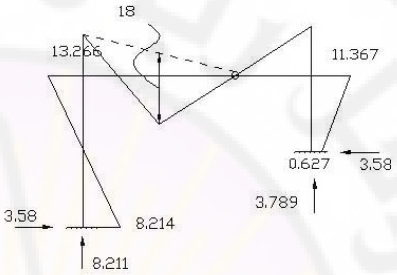




مسألة (جـ) :



$$\delta_{10} = -\frac{1}{2}6(6)36 = -648$$



$$\delta_{20} = \frac{1}{2}3(36)5 + 6(36)6 = 1566$$

$$\delta_{11} = \frac{1}{2}6(6)\frac{2}{3}6 + \frac{1}{2}3(3)\frac{2}{3}3 = 81$$

$$\delta_{12} = -\frac{1}{2}6(6)6 + \frac{1}{2}3(3)3 = -94.5$$

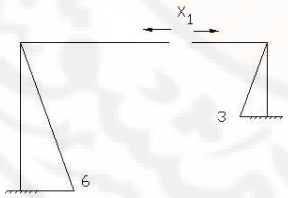
$$\delta_{22} = \frac{1}{2}6(6)4 + 6(6)6 + \frac{1}{2}3(3)2 + 3(3)3 = 324$$

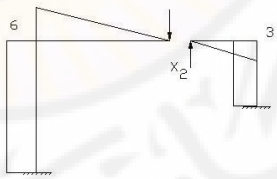
$$\begin{bmatrix} 81 & -94.5 \\ -94.5 & 324 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 648 \\ -1566 \end{bmatrix}$$

$$213.75X_2 = -810$$

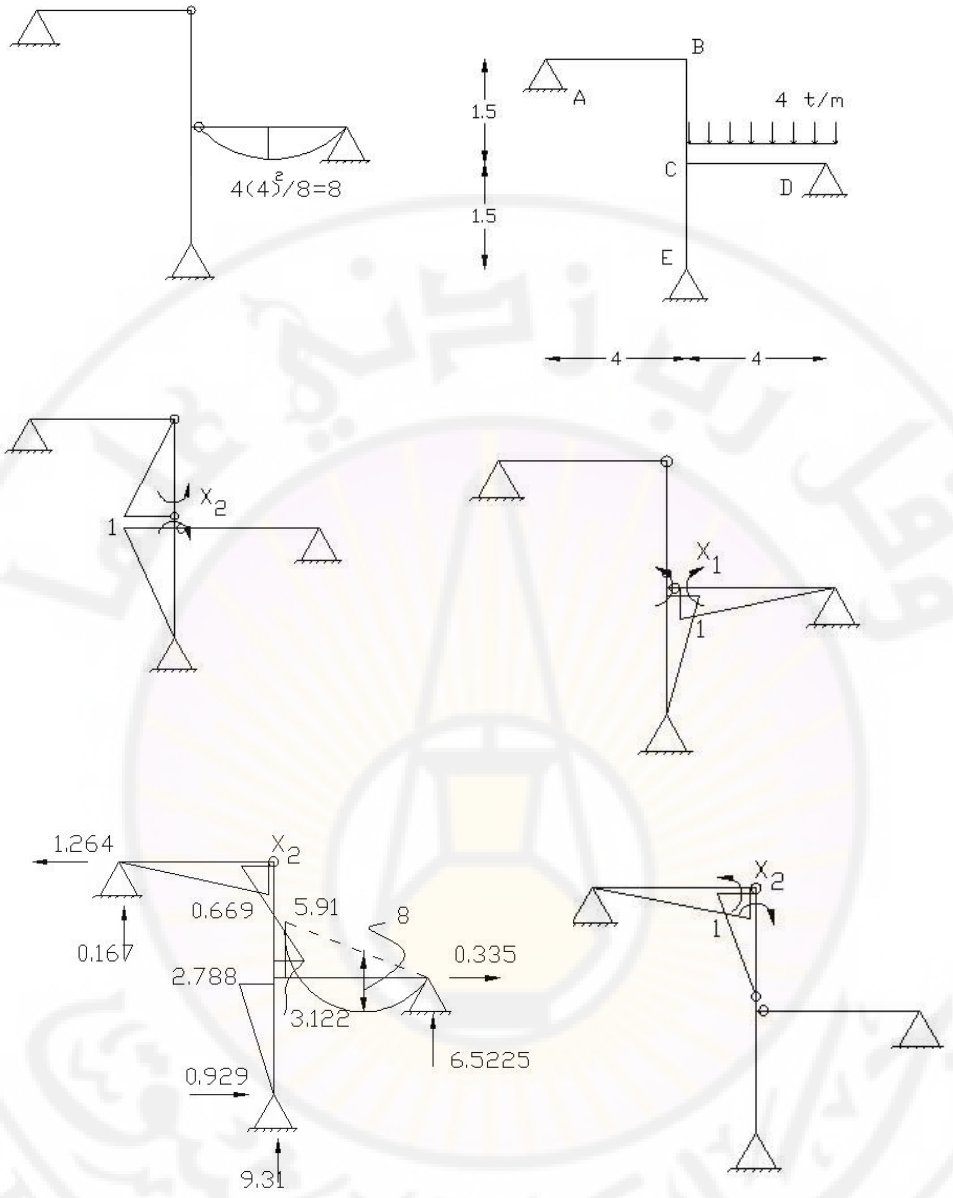
$$X_1 = -3.789$$

$$X_2 = +3.580$$





مسألة (د) :


4(4)²/8=8
B
A
4 t/m
C
D
E
1.5
1.5
4
4
X2
1
X1
1
1.264
X2
0.669
5.91
8
0.167
0.335
2.788
3.122
6.5225
0.929
9.31
X2
1

$$\delta_{10} = \frac{2}{3}4(8)\frac{1}{2} = 10.667$$

$$\delta_{20} = 0$$

$$\delta_{30} = 0$$

$$\delta_{11} = \frac{1}{2}4\frac{2}{3} + \frac{1}{2}3\frac{2}{3} = 2.333$$

$$\delta_{12} = -\frac{1}{2}3\frac{2}{3} = -1$$

$$\delta_{13} = 0$$

$$\delta_{22} = \frac{1}{2}3\frac{2}{3}2 = 2$$

$$\delta_{23} = \frac{1}{2}3\frac{1}{3} = 0.5$$

$$\begin{bmatrix} 2.333 & -1 & 0 \\ -1 & 2 & 0.5 \\ 0 & 0.5 & 2.333 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -10.667 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2.333 & -1 & 0 \\ 0 & 1.571 & 0.5 \\ 0 & 0 & 2.174 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -10.667 \\ -4.571 \\ 1.4548 \end{bmatrix}$$

$$X_3 = 0.669$$

$$X_2 = -3.122$$

$$X_1 = -5.91$$

مسألة (هـ) :

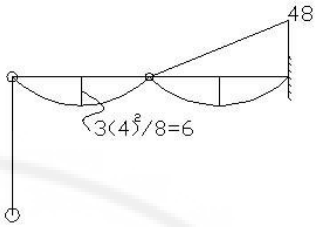
$$\delta_{10} = \frac{1}{2}4(48)\frac{2}{3} = 64$$

$$\delta_{20} = 0$$

$$\delta_{11} = \frac{1}{2}4\frac{2}{3}3 = 4$$

$$\delta_{12} = \frac{1}{2}4\frac{1}{3} = 0.667$$

$$\delta_{22} = \frac{1}{2}4\frac{2}{3} = 1.333$$

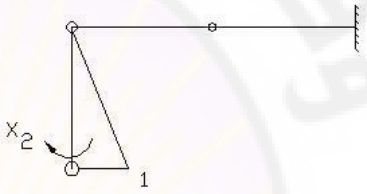


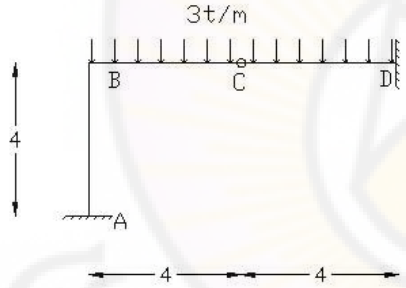
$$\begin{bmatrix} 4 & 0.667 \\ 0.667 & 1.333 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -64 \\ 0 \end{bmatrix}$$

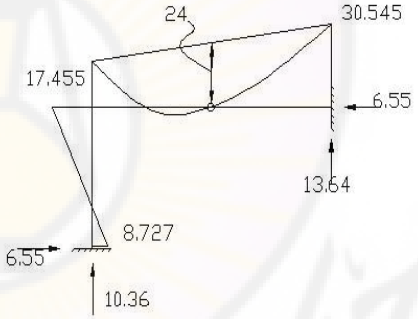
$$1.222X_2 = 10.667$$

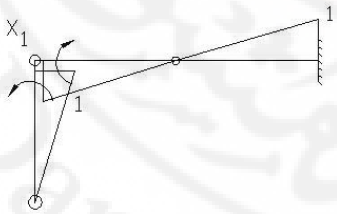
$$X_2 = 8.727$$

$$X_1 = -17.455$$

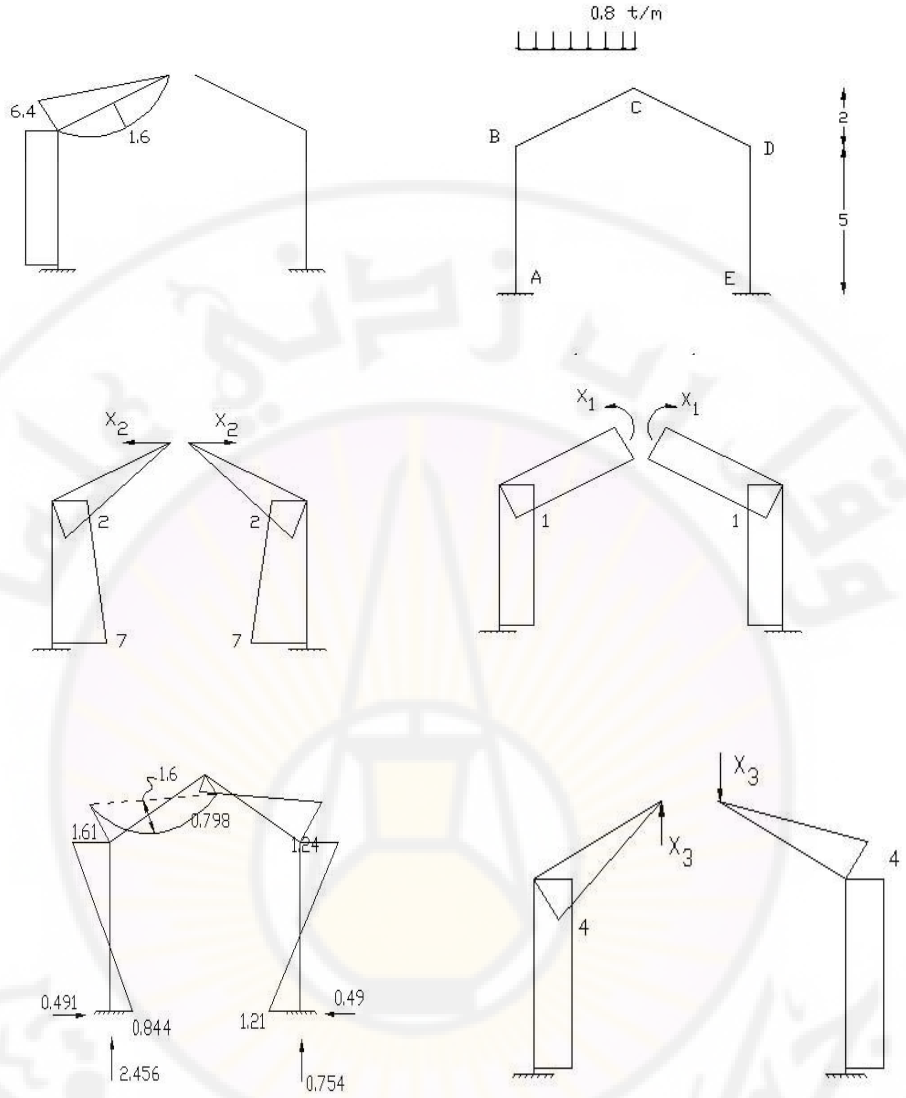








مسألة (و) :


0.8 t/m
6.4
1.6
B
C
D
A
E
2
5
X2
X2
2
2
7
7
X1
X1
1
1
1.6
1.61
0.798
1.24
0.491
0.844
2.456
1.21
0.49
0.754
X3
X3
4
4

$$\delta_{10} = \frac{2}{3}1.6(4.472) - \frac{1}{2}6.4(4.472) - 5(6.4) = -41.54$$

$$\delta_{20} = \frac{2}{3}1.6(4.472) - \frac{1}{2}6.4(4.472)\frac{2}{3}2 - \frac{2+7}{2}5(6.4) = -158.31$$

$$\delta_{30} = \frac{2}{3}1.6(4.472)2 - \frac{1}{2}6.4(4.472)\frac{2}{3}4 - 4(5)(6.4) = -156.62$$

$$\delta_{11} = 9.472(2) = 18.944$$

$$\delta_{12} = \left[\frac{1}{2}4.472(2) + \frac{2+7}{2}5\right]2 = 53.944$$

$$\delta_{13} = 0$$

$$\delta_{22} = \left\{\frac{1}{2}4.472(2)\frac{2}{3} + 2 + \frac{5}{6}[2(2)2 + 2(7)7 + 2(2)7]\right\}2 = 235.26$$

$$\delta_{23} = 0$$

$$\delta_{33} = \left[\frac{1}{2}4.472(4)\frac{2}{3}4 + 5(4)4\right]2 = 207.7$$

$$\begin{bmatrix} 18.944 & 53.944 & 0 \\ 53.944 & 235.26 & 0 \\ 0 & 0 & 207.7 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 41.54 \\ 158.31 \\ 156.62 \end{bmatrix}$$

$$\begin{bmatrix} 18.944 & 53.944 & 0 \\ 0 & 81.65 & 0 \\ 0 & 0 & 207.7 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 41.54 \\ 40.02 \\ 156.62 \end{bmatrix}$$

$$X_3 = 0.754$$

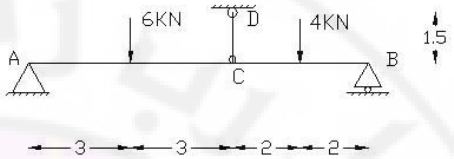
$$X_2 = 0.49$$

$$X_1 = 0.798$$

مسألة (ز) :

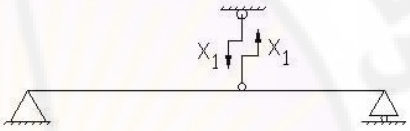
$$E_t I\delta_{10} = -\frac{1}{2}3(15)\frac{2}{3}1.2 - \frac{3}{6}[2(15)(1.2)+15(2.4)+2(12)(2.4)+12(1.2)]$$
$$-\frac{2}{6}[2(12)(2.4)+12(1.2)+10(2.4)+2(10)(1.2)]-\frac{1}{2}2(10)\frac{2}{3}1.2$$
$$= -18-\frac{1}{2}[36+36+57.6+14.4]-\frac{1}{3}[57.6+14.4+24+24]-8$$
$$= -18-72-40-8$$

$$\delta_{10} = -138/E_t I$$



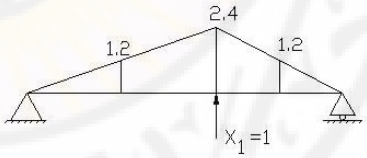
$$\delta_{11} = \int \frac{M_1^{\,2}}{E_t I}dL + \int \frac{N_1^{\,2}}{E_S A}dL$$
$$= \frac{1}{2}6(2.4)\frac{2}{3.}2.4\frac{1}{E_t I} + \frac{1.5}{E_S A}$$

$$\delta_{10} + X_1\delta_{11} = 0$$



$$-\frac{138}{E_t I} + X_1\left(\frac{19.2}{E_t I} + \frac{1.5}{E_S A}\right) = 0$$
$$-138 + X_1\left(19.2 + \frac{E_t I_t}{E_S A_S}1.5\right) = 0$$
$$-138 + X_1\left(19.2 + \frac{10^7 \times 0.15\frac{(0.2)^3}{12}}{2\times10^8 \times 150\times10^{-6}}1.5\right) = 0$$
$$-138 + X_1(19.2+0.05) = 0$$
$$X_1 = \frac{138}{19.25} = 7.1688kN$$

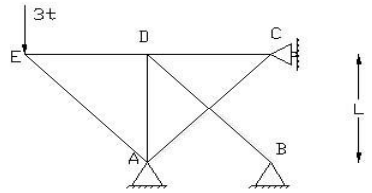
M1

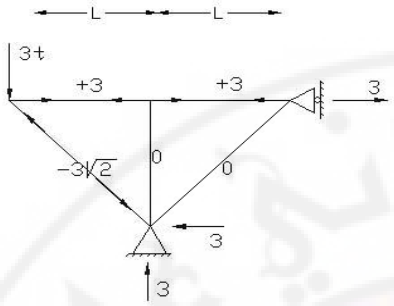


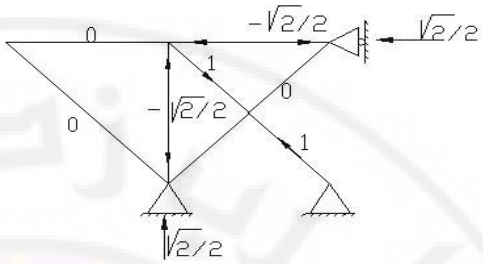
N1



مسألة (حـ) :







القضيب	N_0	N_1	N
ED	+3	0	+3
DC	+3	$-\sqrt{2}/2$	+2.3787
AE	$-3\sqrt{2}$	0	$-3\sqrt{2}=-4.243$
AD	0	$-\sqrt{2}/2$	-0.6132
AC	0	0	0
DB	0	1	+0.8787

$$\delta_{10}=\sum N_0 N_1 L$$

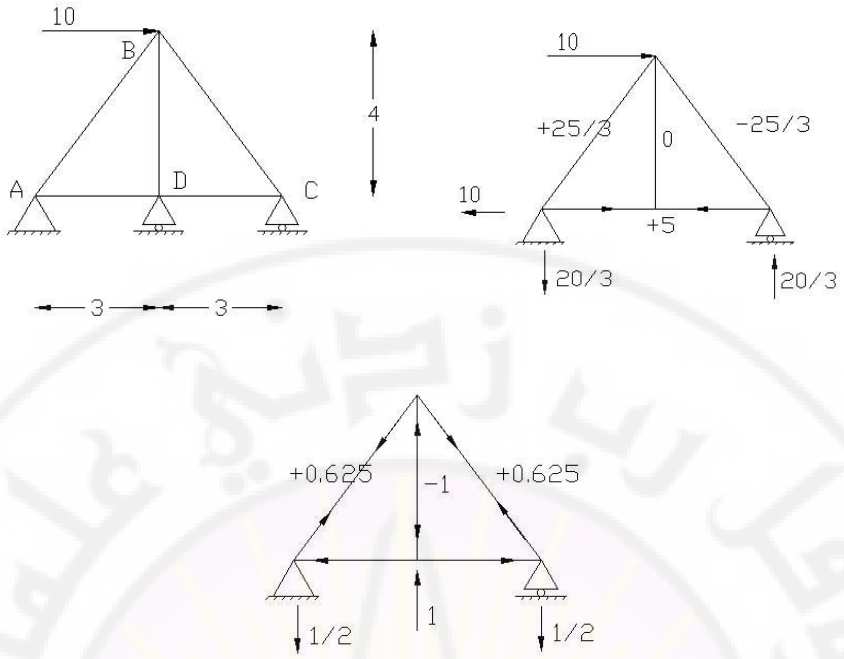
$$=-3\frac{\sqrt{2}}{2}6=-9\sqrt{2}$$

$$\delta_{11}=\left[\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}6+\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}6+1\left(6\sqrt{2}\right)\right]$$

$$=\frac{9\sqrt{2}}{6\left(1+\sqrt{2}\right)}=\frac{3\sqrt{2}}{2\left(1+\sqrt{2}\right)}=0.8787$$

$$N=N_0+XN_1$$

مسألة (ط) :



القضيب	N_0	N_1	L	N
AB	+25/3	+0.625	5	+9.137
AD	+5	-0.375	3	+4.518
BC	-25/3	+0.625	5	-7.530
BD	0	-1	4	-1.286
CD	+5	-0.375	3	+4.518

$$\delta_{10} = [5(-0.375)32] = -11.25$$

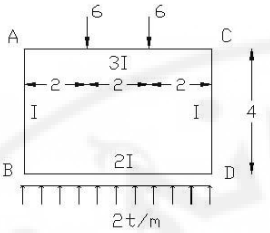
$$\delta_{11} = \left[(0.625)^2 5(2) + (0.375)^2 3(2) + 1(4)\right]$$

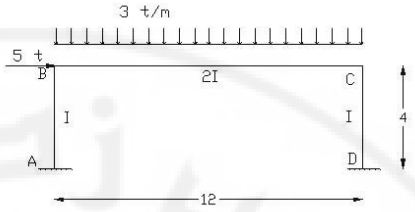
$$= 8.75$$

$$X_1 = 1.2857$$

المركز المرن والتشبيه العمودي

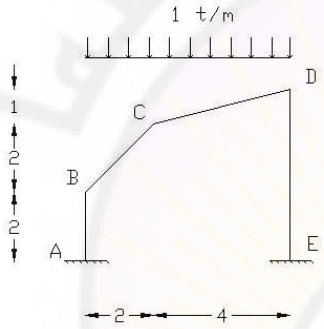
- يطلب حل المنشآت غير المقررة الآتية بطريقة المركز المرن والتشبيه العمودي

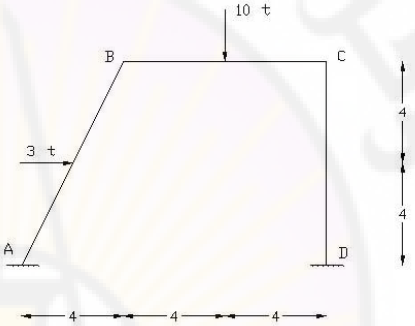




- آ -

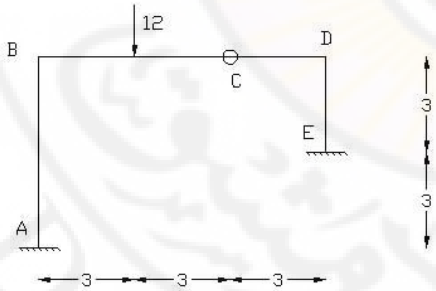
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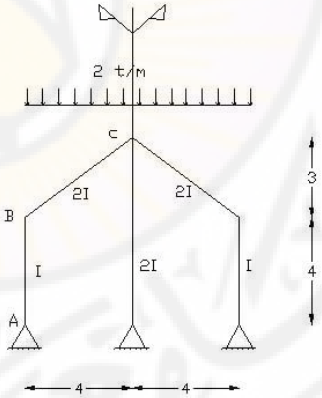




- جـ -

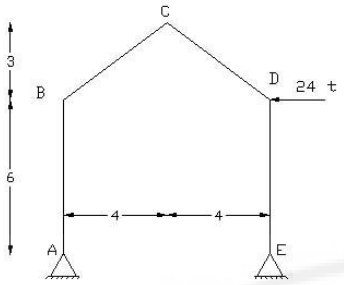
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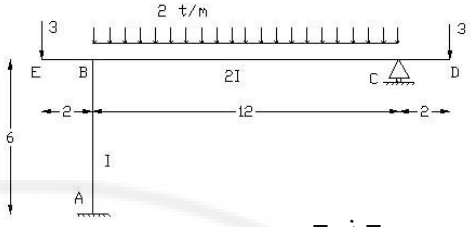


- و -

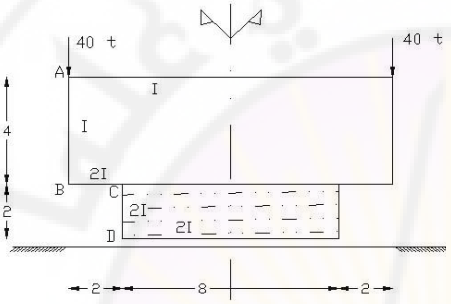
- هـ -



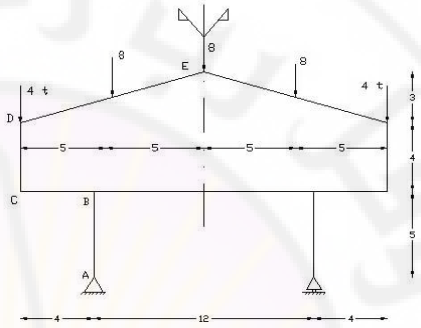
– حـ –



– ز –

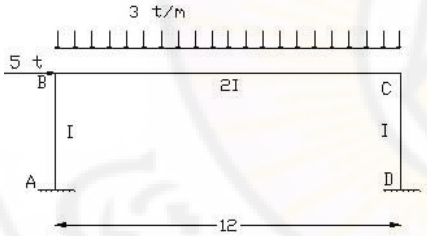


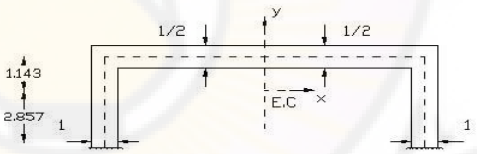
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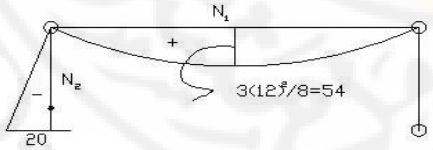


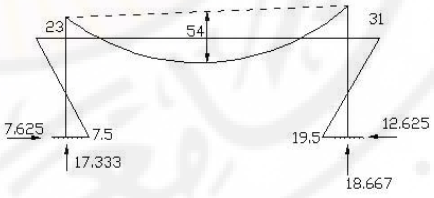
– ك –

مسألة (آ) :









$$\bar{y} = \frac{2(1)4(2) + \frac{1}{2}12(4)}{2(1)4 + \frac{1}{2}12} = \frac{16+24}{14} = \frac{40}{14} = 2.857$$

$$I_x = 2\frac{4^3}{3} - 14(1.143)^2 = 24.376$$

$$I_y = \frac{1}{2}\frac{12^3}{12} + 2(1)4(6)^2 = 360$$

$$N_1 = \frac{1}{2}\frac{2}{3}12(54) = 216$$

$$N_2 = -\frac{1}{2}4(20) = -40$$

$$N = 176$$

$$M_x = +216(1.143) + 40\left(2.857 - \frac{4}{3}\right) = 307.835$$

$$M_y = +40(6) = 240$$

$$X_1 = -\frac{307.835}{24.376} = -12.629$$

$$X_2 = -\frac{240}{360} = -0.667$$

$$X_3 = -\frac{176}{14} = -12.57$$

$$M = M_0 - 12.629y - 0.667x - 12.57$$

مسألة (ب) :

$$\bar{y} = \frac{2(1)4(2) + \frac{1}{3}6(4)}{2(1)4 + \frac{1}{3}6 + \frac{1}{2}6} = \frac{16+8}{8+2+3} = \frac{24}{13} = 1.846$$

$$I_x = 2\frac{4^3}{3} + \frac{1}{3}6(4)^2 - 13(1.846)^2 = 30.366$$

$$N_1 = +\frac{1}{3}4(12) = +16$$

$$N_2 = \frac{1}{2}\frac{2}{3}6(9) = +18$$

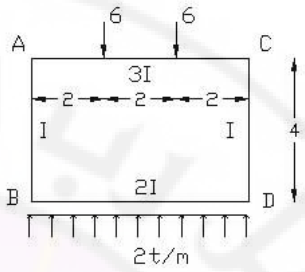
$$N = 34$$

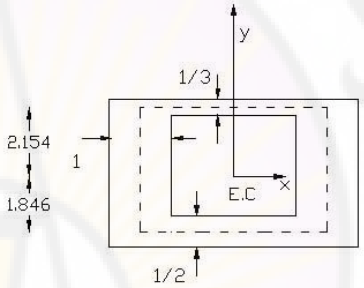
$$M_x = 16(2.154) - 18(21.846) = 1.236$$

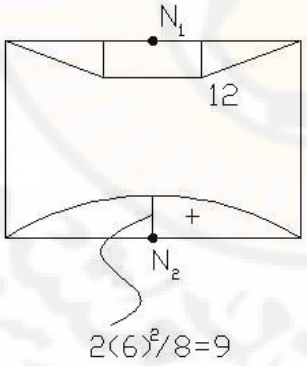
$$X_1 = -\frac{M_x}{I_x} = -\frac{1.236}{30.366} = -0.041$$

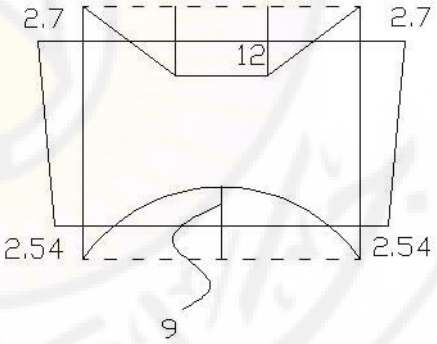
$$X_3 = -\frac{N}{A} = -\frac{34}{13} = -2.615$$

$$M = M_0 - 0.041y - 2.615$$









مسألة (جـ) :

$$x_C = \frac{\sqrt{80}(2)+18(12)+8(8)}{8+8+\sqrt{80}} = \frac{177.89}{24.944} = 7.13$$

$$y_C = \frac{\sqrt{80}(4)+8(4)+8(8)}{24.944} = 5.28$$

$$I_x = \frac{8^3}{3} + \sqrt{80}\,\frac{8^2}{3} - 24.944(2.72)^2 = 176.93$$

$$I_y = \frac{8^3}{12} + 8(0.87)^2 + 8(4.87)^2 + \sqrt{80}\,\frac{4^2}{12} + \sqrt{80}(5.13)^2 = 485.77$$

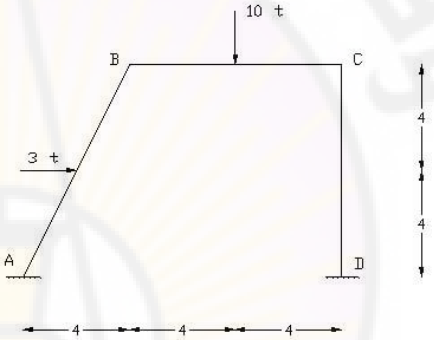
$$I_{xy} = \sqrt{80}\,\frac{4(8)}{12} + \sqrt{80}\,\frac{4^2}{12} + \sqrt{80}(5.13)(1.28) + 8(2.72)(0.87) - 8(4.87)(1.28) = 51.645$$

$$I'_x = Ix\left(1 - \frac{I_{xy}}{I_x I_y}\right)$$

$$= 176.93\left[1 - \frac{51.645^2}{176.93(485.77)}\right]$$

$$= 171.44$$

$$I'_y = I_y\left(1 - \frac{I_{xy}{}^2}{I_x I_y}\right) = 470.69$$



$$N = -\frac{1}{2}\frac{\sqrt{80}}{2}12 - \frac{1}{2}4(40) - 8(40)$$

$$= -26.83 - 80 - 320 = -426.83$$

$$M_x = -26.83(-3.95) - 80(2.72) - 320(-1.28)$$

$$= 297.9$$

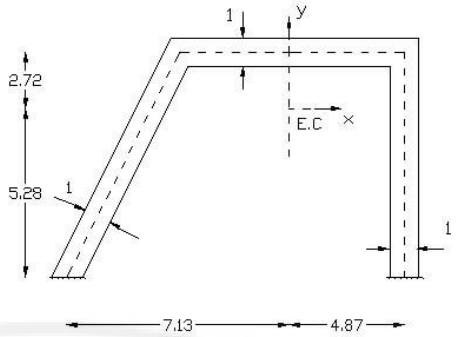
$$M_y = -26.83(-5.8) - 80(3.54) - 320(4.87)$$

$$= -1685.8$$

$$M'_x = M_x - M_y \frac{I_{xy}}{I_y}$$

$$= 297.9 + 1685.8 \frac{51.645}{485.77}$$

$$= 477.13$$



$$M'_y = M_y - M_x \frac{I_{xy}}{I_x}$$

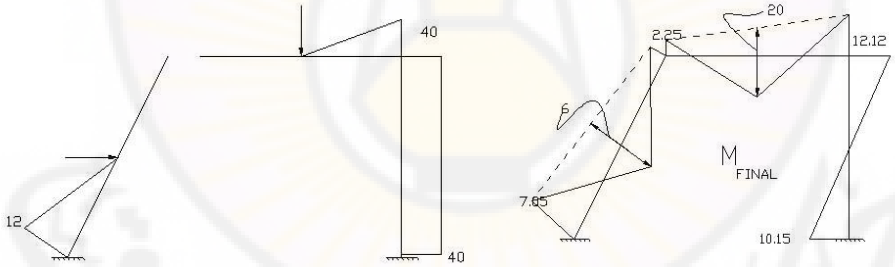
$$= -1685.8 - 297.9 \frac{51.645}{176.93}$$

$$= 1772.8$$

$$M = M_0 - \frac{M'_x}{I'_x} y - \frac{M'_y}{I'_y} x - \frac{N}{A}$$

$$= M_0 - \frac{477.13}{171.44} y + \frac{1772.8}{470.69} x + \frac{426.83}{24.944}$$

$$= M_0 - 2.783y + 3.766x + 17.112$$



مسألة (د) :

$$x_C = \frac{\sqrt{80}(1)+\sqrt{71}(4)+5(6)}{2+\sqrt{8}+\sqrt{17}+5} = \frac{49.321}{13.951} = 3.54$$

$$y_C = \frac{2(1)+\sqrt{80}(3)+\sqrt{148}(4.5)+5(2.5)}{13.951} = 2.98$$

$$I_x = \frac{2^3}{12} + 2(1.98)^2 + \sqrt{8}\frac{2^2}{12} + \sqrt{8}(0.02)^2 + \sqrt{17}\frac{1^2}{12} + \sqrt{17}(1.52)^2 + \frac{5^3}{12} + 5(0.48)^2$$

$$= 30.89$$

$$I_y = 2(3.54)^2 + \sqrt{8}\frac{2^2}{12} + \sqrt{8}(2.54)^2 + \sqrt{17}\frac{4^2}{12} + \sqrt{17}(0.46)^2 + 5(2.46)^2$$

$$= 80.88$$

$$I_{xy} = 2(-3.54)(-1.98) + \sqrt{8}\frac{2(2)}{12} + \sqrt{8}(-2.54)(0.02)$$

$$+ \sqrt{17}\frac{1(4)}{12} + \sqrt{17}(0.46)(1.52) + 5(2.46)(-0.48)$$

$$= 13.17$$

$$I'_x = I_x\left(1 - \frac{I^2{}_{xy}}{I_x I_y}\right)$$

$$= 30.89\left[1 - \frac{13.17^2}{30.89(80.88)}\right]$$

$$= 28.75$$

$$I'_y = I_y\left(1 - \frac{I^2{}_{xy}}{I_x I_y}\right) = 75.26$$

$$N = \frac{1}{2}2(8) + \frac{2}{3}\sqrt{8}(0.5) + \frac{2}{3}\sqrt{17}(2) + \frac{1}{2}5(20)$$

$$= 8 + 0.9428 + 5.4975 + 50 = 64.44$$

$$M_x = 8(-2.31) + 0.9428(0.02) + 5.4975(1.52) + 50(-1.31)$$

$$= -75.6$$

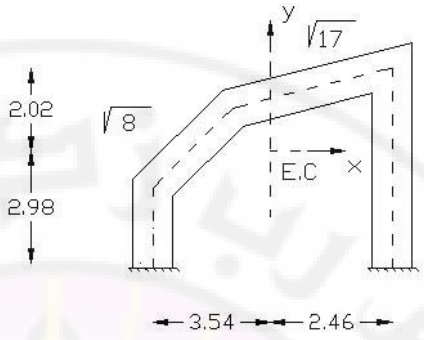
$$M_y = 8(-3.54) + 0.9428(-2.54) + 5.4975(0.46) + 50(2.46)$$

$$= 94.8$$

$$M'_x = M_x - M_y \frac{I_{xy}}{I_y}$$

$$= -75.6 - 94.8\frac{13.17}{80.88}$$

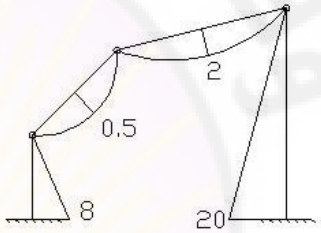
$$= -91.04$$



$$M'_y = M_y - M_x \frac{I_{xy}}{I_x}$$

$$= 94.8 + 75.6\frac{13.17}{30.89}$$

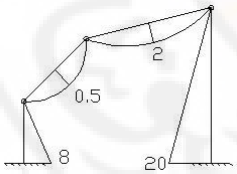
$$= 127$$

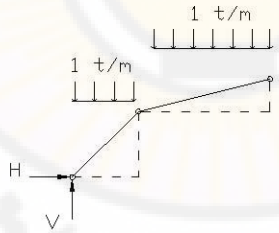


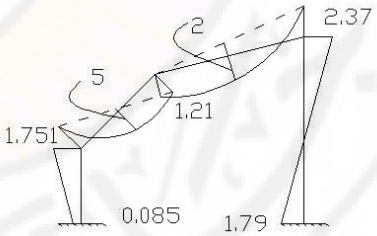
$$M = M_0 - \frac{M'_x}{I'_x} y - \frac{M'_y}{I'_y} x - \frac{N}{A}$$

$$= M_0 - \frac{91.04}{28.75} y + \frac{127}{75.26} x - \frac{64.44}{13.951}$$

$$= M_0 + 3.167y - 1.687x - 4.619$$







مسألة (هـ) :

$$A = \infty$$

$$I_x = \frac{1(4)^3}{3} + \frac{1}{2}(5)\frac{3^2}{12} + \frac{1}{2}(5)(5.5)^2$$

$$= 98.83$$

$$I_y = \frac{1}{2}(5)\frac{4^2}{3}$$

$$= 13.33$$

$$I_{xy} = \frac{1}{2}(5)\frac{3(4)}{12} + \frac{1}{2}(5)(2)(5.5)$$

$$= 30$$

$$I'_x = I_x\left(1 - \frac{I^2_{xy}}{I_x I_y}\right)$$

$$= 98.83\left[1 - \frac{30^2}{98.83(13.33)}\right]$$

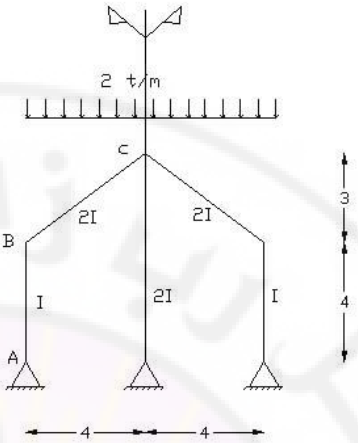
$$= 31.33$$

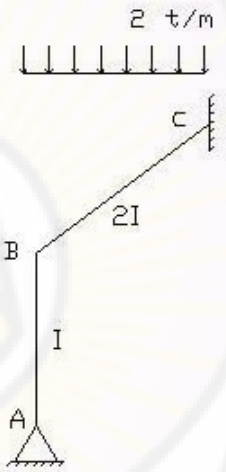
$$I'_y = I_y\left(1 - \frac{I^2_{xy}}{I_x I_y}\right) = 4.23$$

$$N = \frac{1}{2}\frac{2}{3}5(4) = 6.67$$

$$M_x = 6.67(5.5) = 36.67$$

$$M_y = 6.67(2) = 13.33$$





$$M'_x = M_x - M_y \frac{I_{xy}}{I_y}$$

$$= 36.67 - 13.33 \frac{30}{13.33}$$

$$= 6.67$$

$$M'_y = M_y - M_x \frac{I_{xy}}{I_x}$$

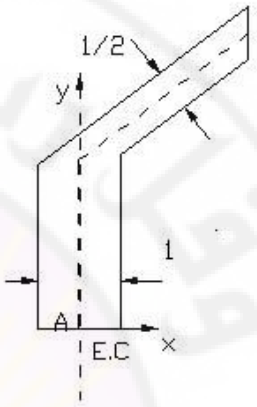
$$= 13.33 + 36.67 \frac{30}{98.83}$$

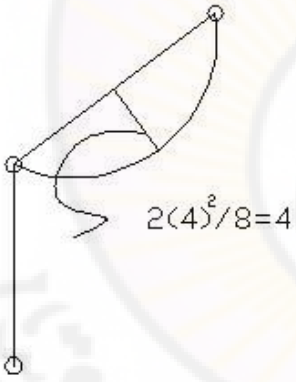
$$= 2.203$$

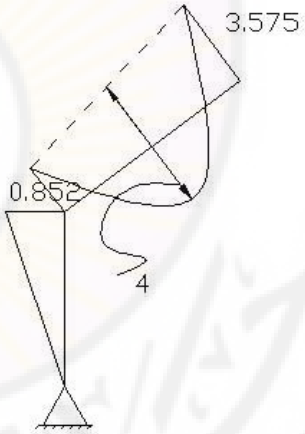
$$M = M_0 - \frac{M'_x}{I'_x} y - \frac{M'_y}{I'_y} x - \frac{N}{A}$$

$$= M_0 - \frac{6.67}{31.33} y + \frac{2.203}{4.23} x$$

$$= M_0 - 0.213y - 0.521x$$







مسألة (و) :

$$A=\infty$$

$$I_x=\frac{1(3)^3}{3}+\frac{1(6)^3}{3}=9+72=81$$

$$I_y=\frac{1(3)^3}{3}+\frac{1(6)^3}{3}+3(3)^2+6(6)^2$$

$$=9+72+27+216=324$$

$$I_{xy}=3(-1.5)(3)+(-6)(-3)$$

$$=94.5$$

$$I'_x=I_x\left(1-\frac{I^2{}_{xy}}{I_xI_y}\right)$$

$$=81\left[1-\frac{94.5^2}{81(324)}\right]=53.44$$

$$I'_y=I_y\left(1-\frac{I^2{}_{xy}}{I_xI_y}\right)=213.75$$

$$N_1=-\frac{1}{2}3(36)=-54$$

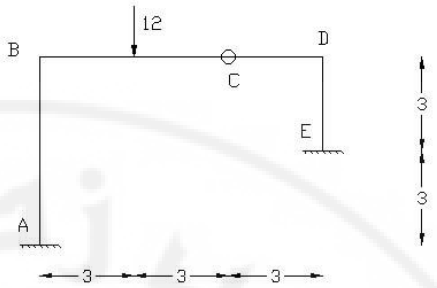
$$N_2=-6(36)=-216$$

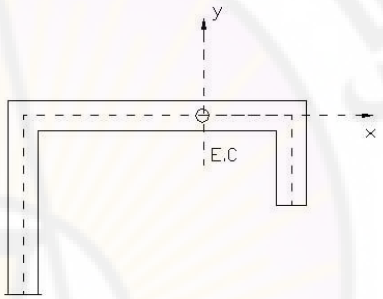
$$M_x=216(-3)=+648$$

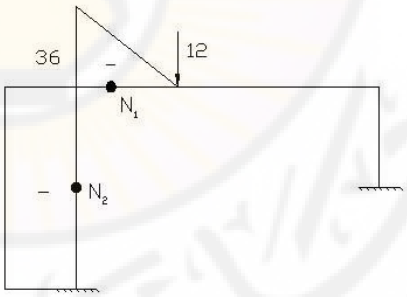
$$M_y=-54(-5)-216(-6)=1566$$

$$M'_x=M_x-M_y\frac{I_{xy}}{I_y}$$

$$=648-1566\frac{94.5}{324}=191.25$$







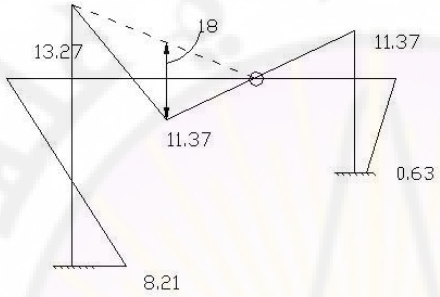
$$M'_y = M_y - M_x \frac{I_{xy}}{I_x}$$

$$= 1566 - 648\frac{94.5}{81} = 810$$

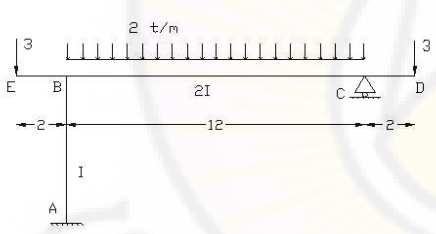
$$M = M_0 - \frac{M'_x}{I'_x} y - \frac{M'_y}{I'_y} x - \frac{N}{A}$$

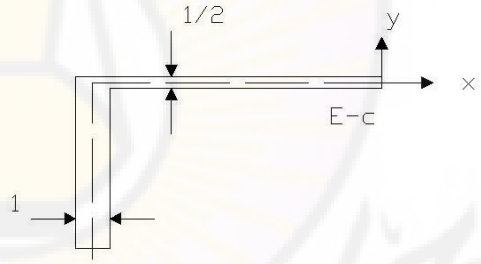
$$= M_0 - \frac{191.25}{53.44} y + \frac{810}{213.75} x$$

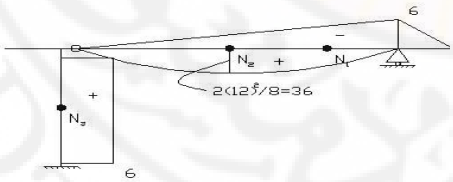
$$= M_0 - 3.579y - 3.689x$$

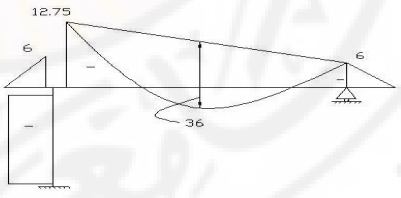


مسألة (ز) :









$$A = \infty$$

$$I_x = \infty$$

$$I_y = \frac{1}{2}\frac{12^3}{3} + 1(6)(12)^2 = 1152$$

$$M_y = -\frac{1}{2}\frac{1}{2}12(6)(-4) + \frac{1}{2}\frac{2}{3}12(36)(-6) + 6(6)(-12)$$

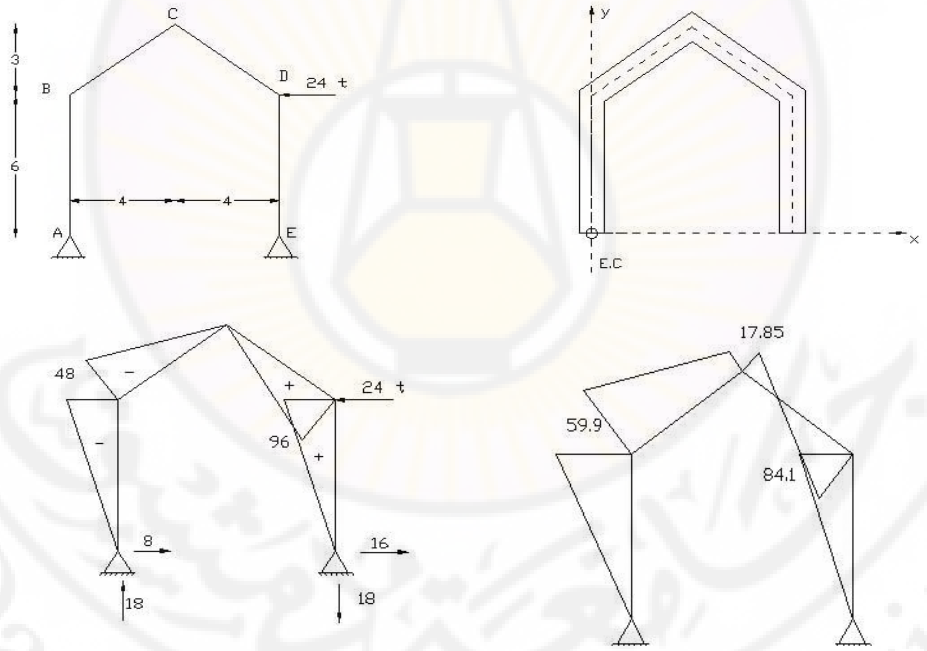
$$= -1224$$

$$X_1 = \frac{-M_y}{I_y}$$

$$= \frac{1224}{1152} = 1.0625$$

$$M = M_0 + 1.0625x$$

مسألة (حـ) :



$$A = \infty$$

$$I_y = \infty$$

$$I_x = 2\left[\frac{6^3}{3} + 5\frac{3^2}{12} + 5(7.5)^2\right]$$

$$= 714$$

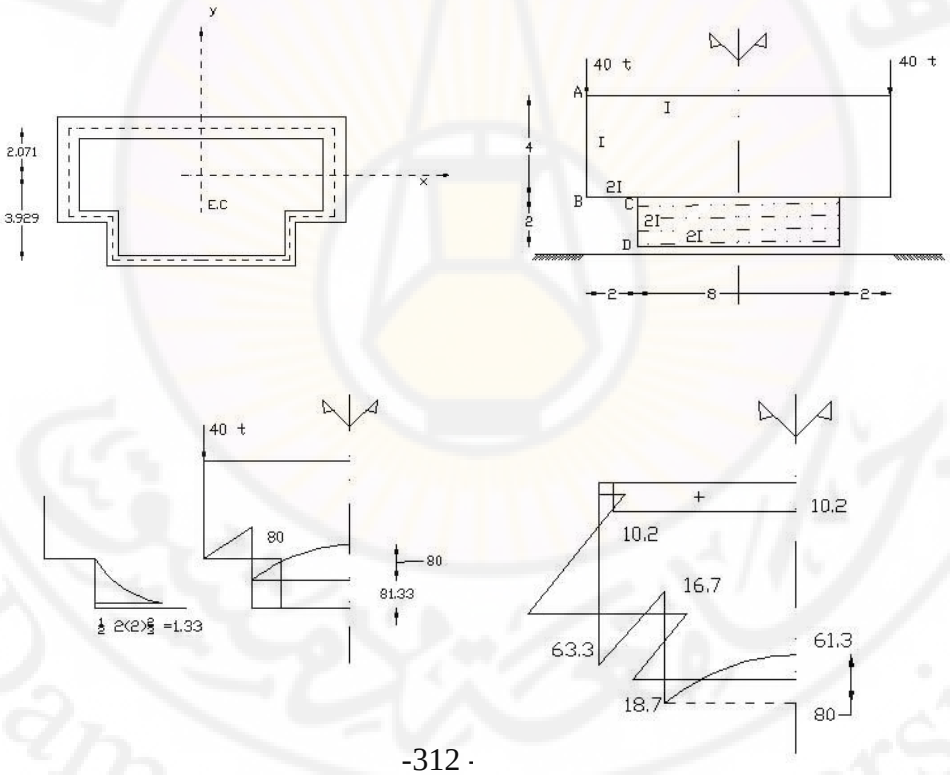
$$M_x = -\frac{1}{2}6(48) + \frac{1}{2}6(96)4 - \frac{1}{2}5(48)7 + \frac{1}{2}5(96)7$$

$$= -1416$$

$$M = M_0 - \frac{1416}{714}y$$

$$= M_0 - 1.983y$$

مسألة (ط) :



$$A = 8\left(\frac{1}{2}\right) + 12(1) + 2\left[4(1) + 2\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right)\right]$$

$$= 4 + 12 + 12 = 28$$

$$y_C = \frac{2\left[2\left(\frac{1}{2}\right)1 + 2\left(\frac{1}{2}\right)2 + 4(1)4\right] + 12(1)6}{28}$$

$$= \frac{110}{28} = 3.929$$

$$I_x = 8\left(\frac{1}{2}\right)(3.929)^2 + 12(1)(2.071)^2$$

$$+ 2\left[2\left(\frac{1}{2}\right)(2.929)^2 + 2\left(\frac{1}{2}\right)(1.929)^2 + 4(1)(0.071)^2 + \frac{1}{2}\frac{2^3}{12} + 1\frac{4^3}{12}\right]$$

$$= 149.19$$

$$N = \left[\frac{1}{2}\left(\frac{1}{2}\right)2(80) + \frac{1}{2}2(80) + \frac{1}{2}\frac{1}{4}2(1.33)\right] + 2\frac{1}{2}8(81.33) + \frac{1}{2}\frac{2}{3}8(80)$$

$$= [40 + 80 + 0.33]2 + 325.32 + 213.33$$

$$= 779.31$$

$$M_x = 2[40(-1.929) + 80(-2.929) + 0.33(-3.529)]$$

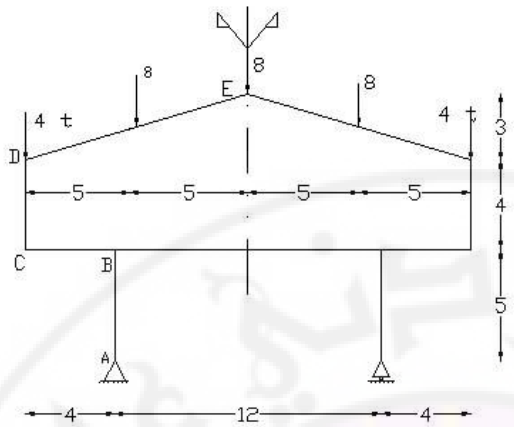
$$+ 325.32(-3.929) + 213.33(-3.929)$$

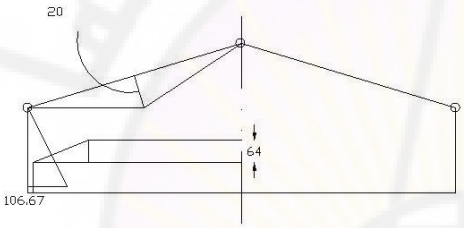
$$= -2741.6$$

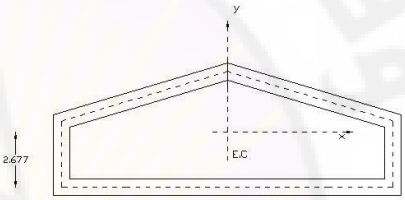
$$M = M_0 + \frac{2741.6}{149.19}y - \frac{779.31}{28}$$

$$= M_0 + 18.38y - 27.83$$

مسألة (ك) :

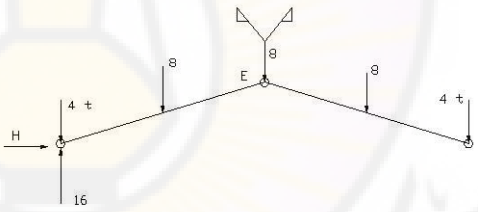






$$y_C = \frac{2(4)2 + 2\sqrt{109}(5.5)}{20 + 2(4) + 2\sqrt{159}}$$

$$= \frac{130.843}{48.88} = 2.677$$



$$A = 48.88$$

$$I_x = 20(2.677)^2 + 2\left[\frac{4^3}{12} + 4(0.677)^2\right] + 2\left[\sqrt{109}\,\frac{3^2}{12} + \sqrt{109}(2.823)^2\right]$$

$$= 339.7$$

$$N = 2\left[\frac{1}{2}\sqrt{109}(20) + \frac{1}{2}4(106.67) + 106.67(10) + \frac{1}{2}4(64) + 6(64)\right]$$

$$= 3792.886$$

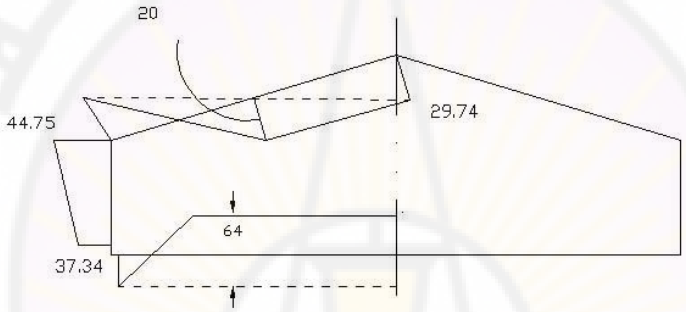
$$M_x = 2\left[\frac{1}{2}\sqrt{109}(20)(2.823) - \frac{1}{2}4(106.67)(1.344) - 106.67(10)(2.677)\right.$$

$$\left. - \frac{1}{2}4(64)(2.677) - 6(64)(2.677)\right]$$

$$= -8436.358$$

$$M = M_0 - \frac{M_x}{I_x}y - \frac{N}{A}$$

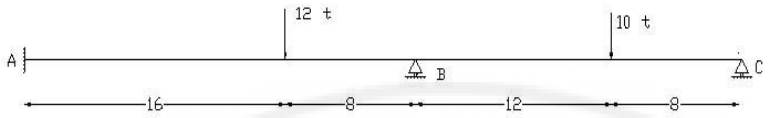
$$= M_0 + 24.83y - 77.60$$



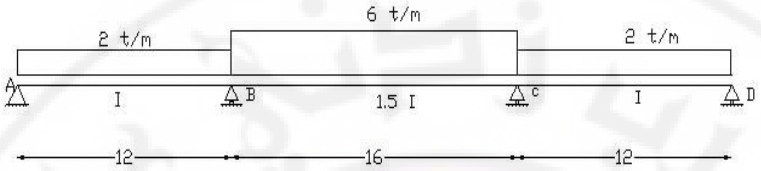
نظريــة العـزوم الثلاثـة

- يطلب حل المنشآت الآتية باستخدام نظرية العزوم الثلاثة :

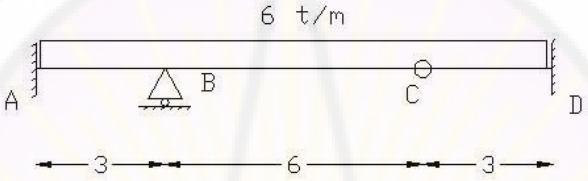
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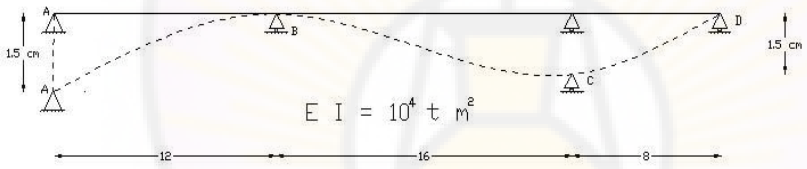
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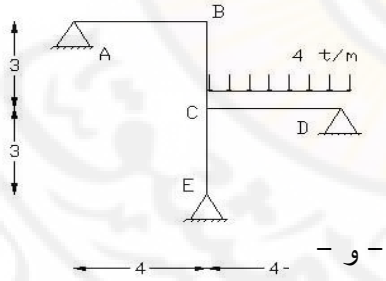
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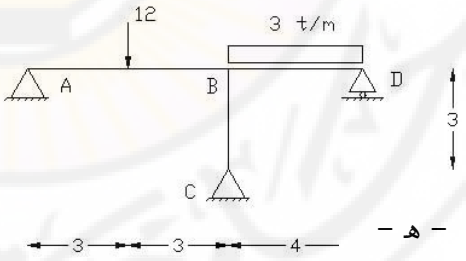
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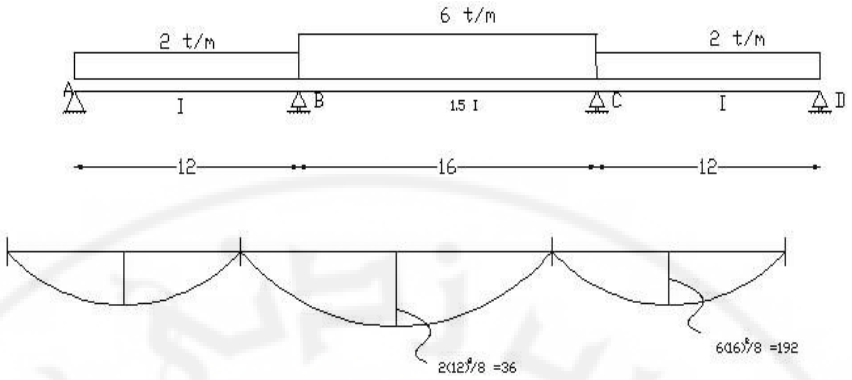
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مسألة (آ) :



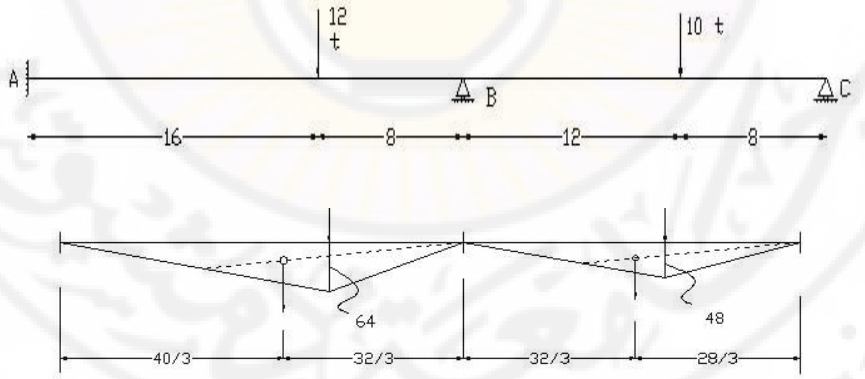
$$2M_B\left(12+\frac{16}{1.5}\right)+M_B\left(\frac{16}{1.5}\right)=$$

$$-6\left[\frac{2}{3}12(36)\right]\frac{1}{2}-6\left[\frac{2}{3}12(192)\right]\frac{1}{1.5}\frac{1}{2}$$

$$\frac{84}{1.5}M_B=-\frac{7440}{1.5}$$

$$M_B=88.57$$

مسألة (ب) :



$$1/\quad 2M_A(24)+M_B(24)=-6\left[\frac{1}{2}24(64)\right]\frac{\frac{32}{3}}{24}$$

$$(1)\qquad 2M_A+M_B=-\frac{256}{3}$$

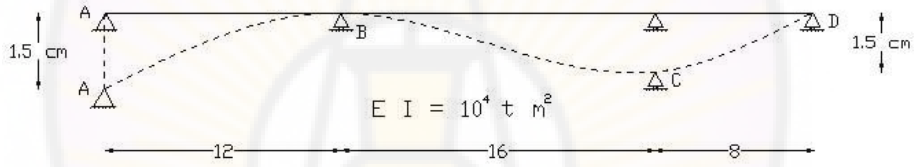
$$2/\quad M_A(24)+2M_B(24+20)=-6\left[\frac{1}{2}24(64)\right]\frac{\frac{40}{3}}{24}-6\left[\frac{1}{2}20(48)\right]\frac{\frac{28}{3}}{20}$$

$$(2)\qquad 24M_A+88M_B=-3904$$

$$\begin{matrix}-4\\ 1\end{matrix}\begin{bmatrix}6 & 3\\ 24 & 80\end{bmatrix}\begin{bmatrix}M_A\\ M_B\end{bmatrix}=\begin{bmatrix}-256\\ -3904\end{bmatrix}$$

$$76M_B=-2880$$
$$M_B=-37.9$$
$$M_A=-23.7$$

مسألة (جـ) :



$$2M_B(12+16)+M_C(16)=-6EI1.5\times10^{-2}\left(\frac{1}{12}+\frac{1}{16}\right)$$

$$(1)\qquad 56M_B+16M_C=-6\times10^4\times1.5\times10^{-2}\,\frac{28}{12\times16}$$

$$=-9\times10^2\,\frac{28}{12\times16}$$

$$=-\frac{2100}{16}=-131.25$$

(2) $16M_B + 48M_C = 9\times10^2 \dfrac{24}{16\times8}$

$$= +168.75$$

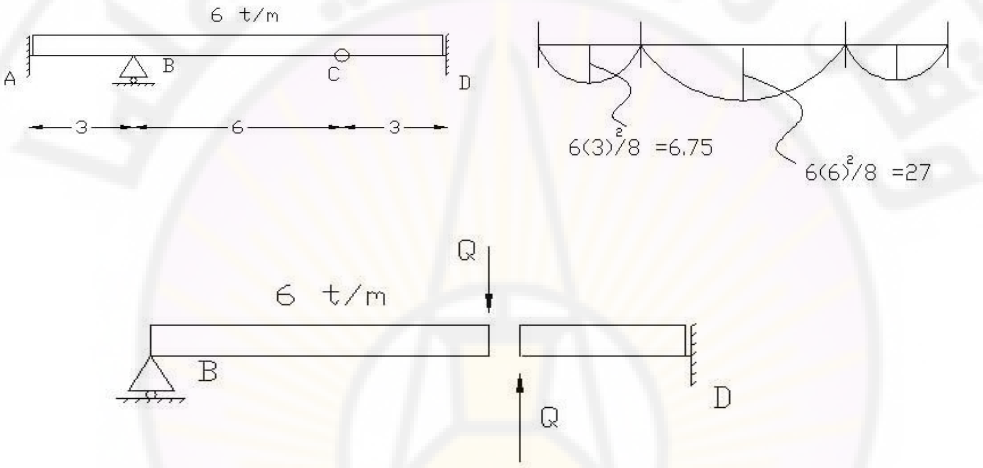
$$\begin{matrix} 3 \\ -1 \end{matrix}\begin{bmatrix} 56 & 16 \\ 16 & 48 \end{bmatrix}\begin{bmatrix} M_B \\ M_C \end{bmatrix} = \begin{bmatrix} -131.25 \\ +168.75 \end{bmatrix}$$

$$152M_B = -562.5$$

$$M_B = -3.70$$

$$M_C = +4.75$$

مسألة (د) :



1/ $2M_A(3) + M_B(3) = -6\left[\dfrac{2}{3}3(6.75)\right]\dfrac{1}{2}$

(1) $2M_A + M_B = -13.5$

2/ $M_A(3) + 2M_B(3+6) = -6\left[\dfrac{2}{3}3(6.75)\right]\dfrac{1}{2} - 6\left[\dfrac{2}{3}6(27)\right]\dfrac{1}{2} - 6EI\dfrac{y}{6}$

(2) $M_A + 6M_B = -121.5 - \dfrac{EIy}{3}$

3/ $2M_D(3) = -6\left[\frac{2}{3}3(6.75)\right]\frac{1}{2} - 6EI\frac{y}{3}$

(3) $2M_D = -13.5 - \frac{2EIy}{3}$

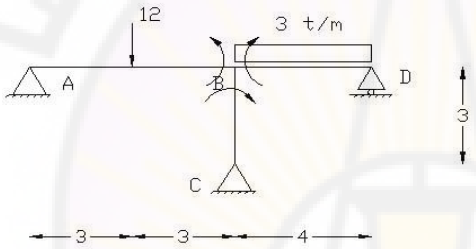
$2(2)-(3) \Rightarrow \quad 2M_A + 12M_B - 2M_D = -229.5$

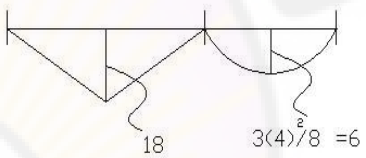
(1) $2M_A \Rightarrow \quad +11M_B - 2M_D = -216$

$$M_B + 2M_D = -162$$

$$12M_B = -378$$
$$M_B = -31.5$$
$$M_D = -65.25$$
$$M_A = +9$$

مسألة (هـ) :





$2M_{BA}(6) + 2M_{BC}(3) = -6\left[\frac{1}{2}6(18)\right]\frac{1}{2}$

(1) $2M_{BA} + M_{BC} = -27$

$2M_{BA}(6) + 2M_{BD}(4) = -6\left[\frac{1}{2}6(18)\right]\frac{1}{2} - 6\left[\frac{2}{3}4(6)\right]\frac{1}{2}$

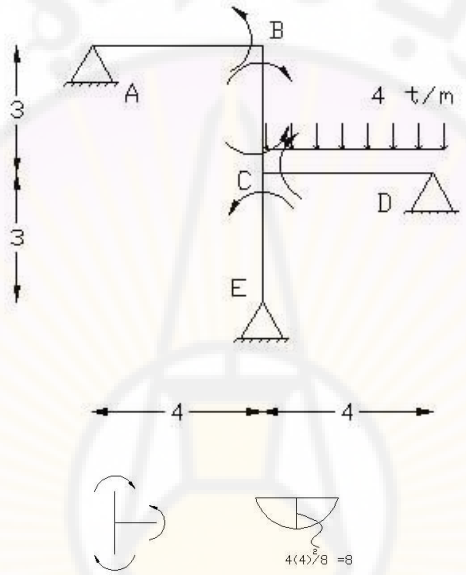
(2) $3M_{BA} + 2M_{BD} = -52.5$

(3) $M_{BA} - M_{BC} - M_{BD} = 0$

$$\begin{matrix} 2 \\ -1 \end{matrix}\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} M_{BA} \\ M_{BC} \end{bmatrix}=\begin{bmatrix} -27 \\ -52.5 \end{bmatrix}$$

$$9M_{BA}=-106.5$$
$$M_{BA}=-11.833$$
$$M_{BC}=-3.333$$
$$M_{BD}=-8.5$$

مسألة (و) :



$$2M_B(4+3)+M_{CB}(3)=0$$

(1) $$14M_B+3M_{CB}=0$$

$$2M_{CE}(3)+2M_{CD}(4)=-6\left[\frac{2}{3}4(8)\right]\frac{1}{2}$$

(2) $$3M_{CE}+4M_{CD}=-32$$

$$M_B(3)+2M_{CB}(3)-2M_{CE}(3)=0$$

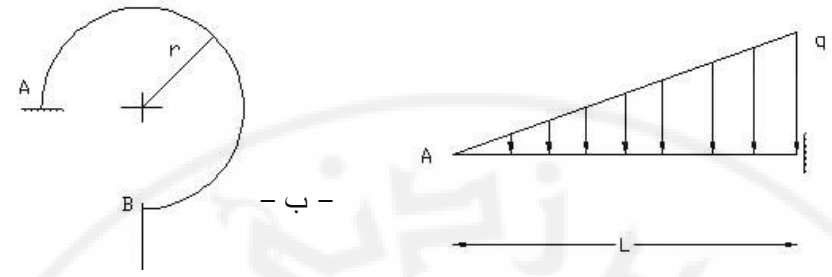
(3) $M_B+2M_{CB}-2M_{CE}=0$

(4) $M_{CE}+M_{CB}-M_{CD}=0$

$$\begin{bmatrix} 14 & 3 & 0 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 3 & 4 \\ 1 & 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} M_B \\ M_{CB} \\ M_{CE} \\ M_{CD} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -32 \\ 0 \end{bmatrix}$$

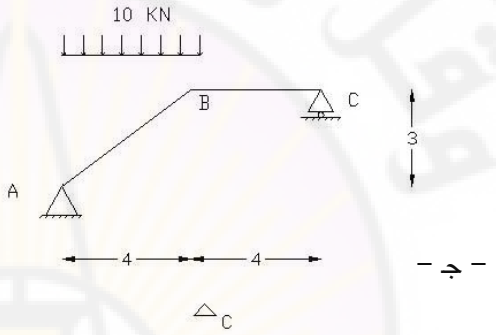
$$M_{CD} = -5.91$$
$$M_{CE} = -2.79$$
$$M_{CB} = -3.12$$
$$M_B = +0.67$$

1- استخدم نظرية كاستليانو لحساب الانتقالات المطلوبة في المنشآت الآتية:

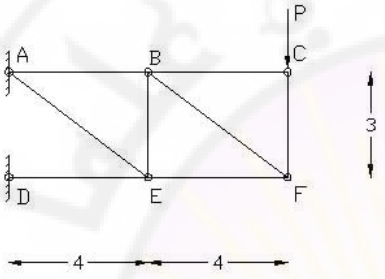


- آ -

- ب -

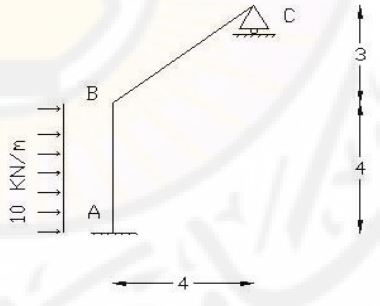


- جـ -

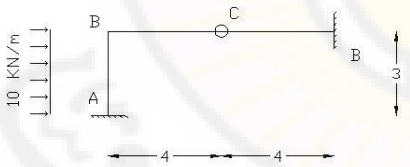


- د -

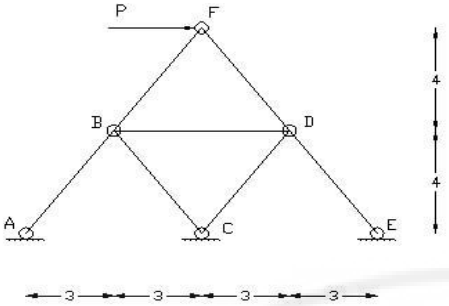
2- استخدم نظرية كاستليانو لحل المنشآت غير المقررة الآتية:



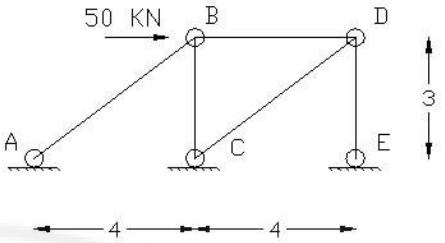
- آ -



- ب -

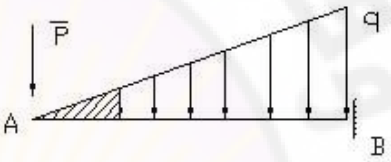


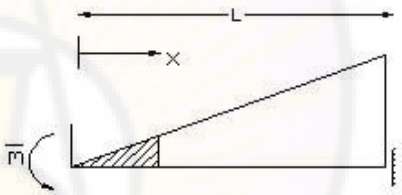
– د –



– جـ –

مسألة (1 – آ) :





$$M_x = \bar{P}x - \frac{1}{2}xq_x\frac{x}{3} \quad , q_x = \frac{q}{L}x$$

$$\frac{\partial M_x}{\partial \bar{P}} = -x$$

$$\Delta_A = \int_0^L \frac{\partial M_x}{\partial \bar{P}}\frac{M_x}{EI}dx$$

$$= \frac{1}{EI}\int_0^L (-x)\left(-\frac{qx^3}{6L}\right)dx = \frac{qL^4}{30EI}$$

$$M_x = -\bar{m} - \frac{1}{2}x\frac{q}{L}x\frac{x}{3}$$

$$\frac{\partial M}{\partial \bar{m}} = -1$$

$$\theta_A = \int_0^L (-1)\left(-\frac{1}{2}\frac{qx^3}{3L}\right)\frac{1}{EI}dx$$

$$= \frac{qL^3}{24EI}$$

مسألة (1 – ب) :

$$M = prSin\theta \quad , \frac{\partial M}{\partial P} = rSin\theta$$

$$V_B = \frac{1}{EI}\int_0^{3\pi/2} (prSin\theta)(rSin\theta)rd\theta$$

$$= \frac{pr^3}{EI}\int_0^{3\pi/2} Sin^2\theta d\theta$$

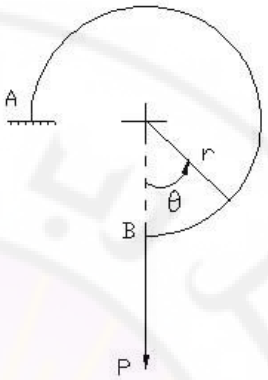
$$= \frac{pr^3}{EI}\int \frac{1}{2}(1-Cos2\theta)d\theta$$

$$= \frac{pr^3}{EI}\frac{1}{2}\left[\theta - \frac{1}{2}Sin2\theta\right]_0^{3\pi/3}$$

$$= \frac{pr^3}{EI}\frac{1}{2}\left[\frac{3\pi}{2}\right]$$

$$= \frac{3\pi}{4}\frac{pr^3}{EI}$$

$$= 2.355\frac{pr^3}{EI}$$



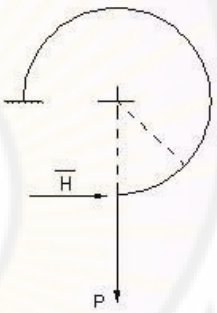
$$M = prSin\theta + \overline{H}(1-Cos\theta)r$$

$$\frac{\partial M}{\partial \overline{H}} = r(1-Cos\theta)$$

$$u_B = \frac{1}{EI}\int (prSin\theta)(1-Cos\theta)r.rd\theta$$

$$= \frac{pr^3}{EI}\int_0 -(1-u)du \quad , u = Cos\theta \quad , du = -Sin\theta d\theta$$

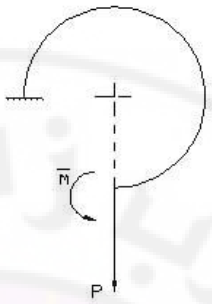
$$= \frac{pr^3}{EI}\left[-Cos\theta + \frac{Cos^2\theta}{2}\right]_0^{3\pi/3}$$



$$= \frac{pr^3}{EI}\left[+1-\frac{1}{2}\right]$$

$$= \frac{1}{2}\frac{pr^3}{EI}$$

$$M = prSin\theta + \overline{m} \quad , \frac{\partial M}{\partial \overline{m}} = 1$$

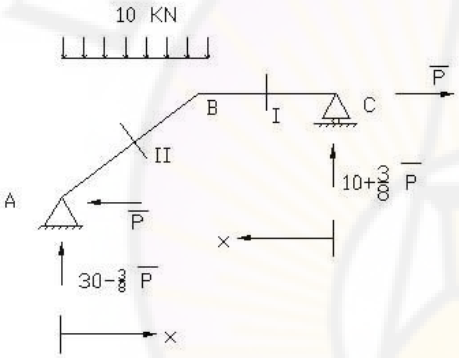


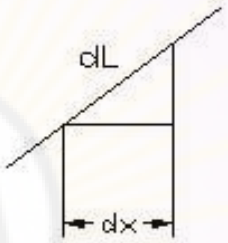
$$\theta_B = \frac{1}{EI}\int_0^{3\pi/2} prSin\theta(1)rd\theta$$

$$= \frac{pr^2}{EI}\left[-Cos\theta\right]_0^{3\pi/3}$$

$$= \frac{pr^2}{EI}$$

مسألة (1 – جـ) :





$$BC: \quad M_I = \left(10+\frac{3}{8}\overline{P}\right)x \quad , \frac{\partial M}{\partial \overline{P}} = \frac{3}{8}x \quad , dL = dx$$

$$AB: \quad M_{II} = \left(30+\frac{3}{8}\overline{P}\right)x - 10\frac{x^2}{2} + \overline{P}\frac{3}{4}x \quad , dL = \frac{5}{4}dx$$

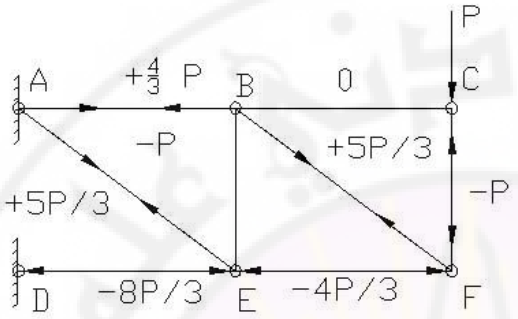
$$\Delta_C = \frac{1}{EI}\int_0^4 \frac{3}{8}x(10x)dx + \frac{1}{EI}\int_0^4 \left(\frac{3}{8}x\right)\left(30x-5x^2\right)\frac{5}{4}dx$$

$$= \frac{1}{EI}\frac{30}{80}\frac{4^3}{3} + \frac{1}{EI}\frac{15}{32}\left(30\frac{4^3}{3} - 5\frac{4^4}{4}\right)$$

$$= \frac{1}{EI}80 + \frac{1}{EI}\frac{15}{32}(10-5)4^3$$

$$= 230\frac{1}{EI}$$

مسألة (1 – د) :



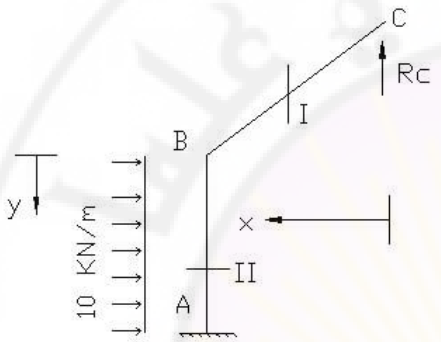
العنصر	N	$\frac{\partial N}{\partial P}$	L
AB	$+4P/3$	4/3	4
AE	$+5P/3$	5/3	5
BC	0	0	4
BE	$-P$	-1	3
BF	$+5P/3$	5/3	5
CF	$-P$	-1	3
DE	$-5P/3$	-8/3	4
EF	$-4p/3$	-4/3	4

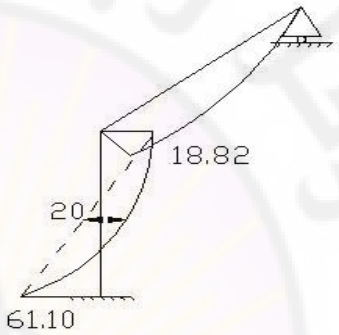
$$\Delta = \sum \frac{\partial N}{\partial P} NL \frac{1}{EA}$$

$$= \left[\frac{4P}{3}\left(\frac{4}{3}\right)4(2) + \frac{5P}{3}\left(\frac{5}{3}\right)5(2) + 0 + (-P)(-1)3(2) + \left(\frac{-8P}{3}\right)\left(\frac{-8}{3}\right)4 \right]$$

$$= \frac{P}{9EA}[128 + 250 + 54 + 256] = \frac{688}{9EA} P = 76.44$$

مسألة (2 – آ) :





$$M_I = R_C x, \quad \frac{\partial M}{\partial R} = x \quad , dL = \frac{5}{4} dx$$

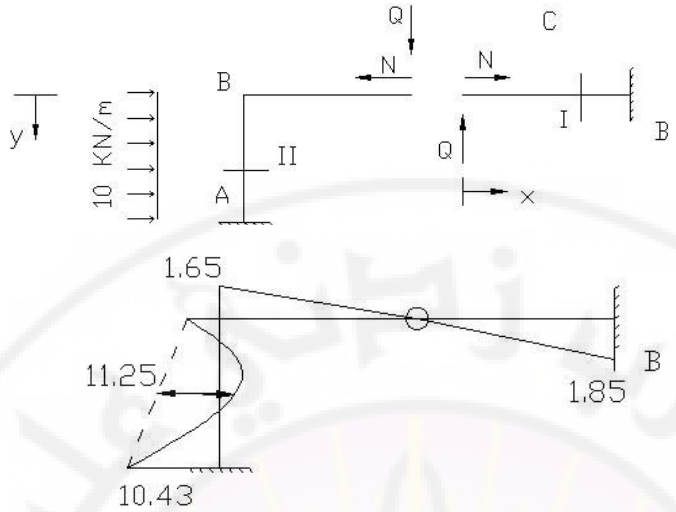
$$M_{II} = 4R_C - 10\frac{y^2}{2}, \quad \frac{\partial M_{II}}{\partial R_C} = 4 \quad , dL = dy$$

$$\frac{\partial M}{\partial R} = 0 = \int_0^4 (R_C x)x \frac{5}{4} dx + \int_0^4 (4R_C - 5y^2)4dy$$

$$\frac{5}{4} R_C \frac{4^3}{3} + 16R_C 4 - 20\frac{4^3}{3} = 0$$

$$17R_C = 80 \Rightarrow R_C = \frac{80}{17} = 4.71$$

مسألة (2 – ب) :



$$M_I = Qx \quad , \frac{\partial M_I}{\partial Q} = x, \frac{\partial M_I}{\partial N} = 0$$

$$M_{II} = Ny - 4Q - 10\frac{y2}{2}$$

$$\frac{\partial M_{II}}{\partial Q} = -4 \quad , \frac{\partial M_{II}}{\partial N} = y$$

1) $$\frac{\partial M}{\partial Q} = 0 = \int_{-4}^{+4} x(Qx)dx + \int_{0}^{3}(-4)(Ny - 4Q - 5y^2)dy$$

$$2Q\frac{4^3}{3} - 4\left[N\frac{3^2}{2} - 4Q(3) - 5\frac{3^3}{3}\right] = 0$$

$$68Q - 13.5N = -135$$

2) $$\frac{\partial M}{\partial N} = 0 = \int_{0}^{3} y(Ny - 4Q - 5y^2)dy$$

$$N\frac{3^3}{3}-4Q\frac{3^2}{3}-5\frac{34}{3}=0$$

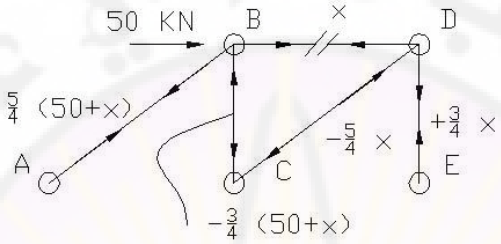
$$-18Q+9N=101.25$$

$$[2(68)-3(18)]\ Q=33.75$$

$$Q=0.4115853$$

$$N=12.07317$$

مسألة (2 – جـ) :



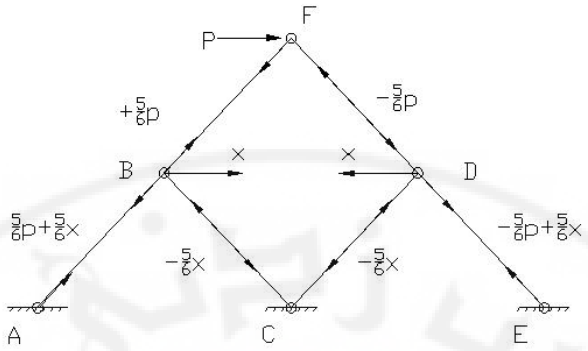
العنصر	N	$\frac{\partial N}{\partial X}$	L	N_{Final}
AB	$(50+X)5/4$	5/4	5	+36.69
BC	$-(50+X)3/4$	-3/4	3	-22.01
BD	X	1	4	-20.65
CD	$-5X/4$	-5/4	5	+25.81
DE	$+3/4X$	3/4	3	-15.49

$$(50+X)\left(\frac{5}{4}\right)^2 5+(50+X)\left(\frac{3}{4}\right)^2 3+X(4)+\left(\frac{5}{4}\right)^2 5+X\left(\frac{3}{4}\right)^2 3=0$$

$$(125+27+64+125+27)X=-(125+27)50$$

$$X=-20.653173$$

مسألة (2 – د) :



العنصر	N	$\frac{\partial N}{\partial X}$	L	N_{Final}
AB	$+5/6P+5/6X$	$5/6$	5	$+5/6P$
BC	$-5/6X$	$-5/6$	5	0
BD	X	1	6	0
BF	$+5/6P$	0	5	$+5/6P$
CD	$-5/6P$	$-5/6$	5	0
DE	$-5/6P+5/6X$	$5/6$	5	$-5/6P$
DF	$-5/6P$	0	5	$-5/6P$

$$\frac{\partial U}{\partial X}=\sum N\frac{\partial N}{\partial X}\frac{L}{EA}$$

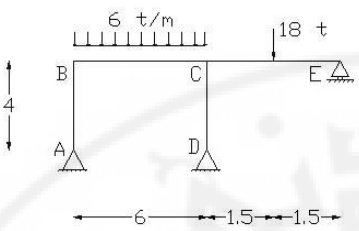
$$\left(\frac{5}{6}\right)^2 5\left[(P+X)+(-P+X)+X(6)\left(\frac{6}{5}\right)^2\frac{1}{5}\right]=0$$

منشأ متناظر عكسياً

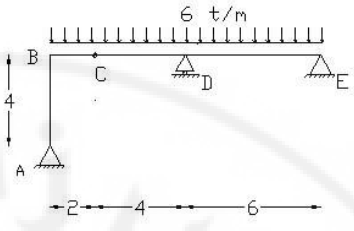
$$X=0$$

مسائل علـى العمـل الوهمـي

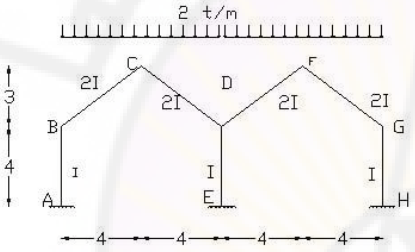
1–يطلب حل المنشآت الآتية باستخدام طريقة العمل الوهمي :



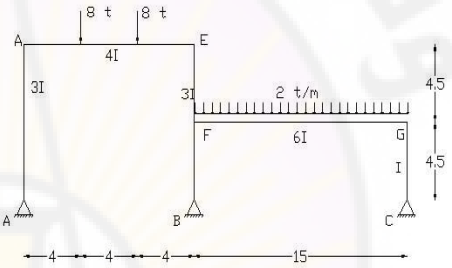
– ب –



– آ –

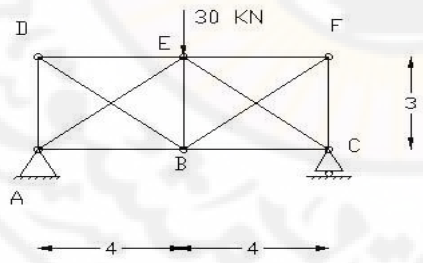


– د –

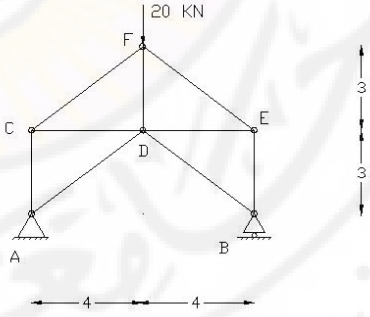


– جـ –

2–يطلب حل الجوائز الشبكية الغير مقررة الآتية :



– ب –



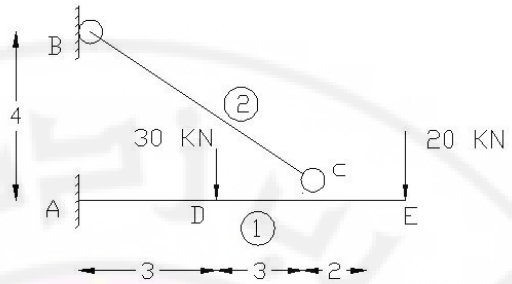
3-يطلب حل المنشأ التالي الذي يحوي عنصر معرض للانعطاف وعنصر معرض لقوى محورية، كما يطلب حساب الانتقالات الشاقولية Δ_D, Δ_E

$E_1 = 2.1\times10^7\ \ KN/m^2$

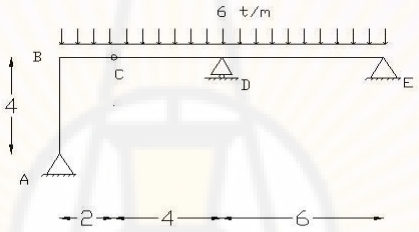
$I_1 = 300000\ \ cm^4$

$E_2 = 2.1\times10^8\ \ KN/m^2$

$A_2 = 4\ \ cm^2$



مسألة (1 – آ) :



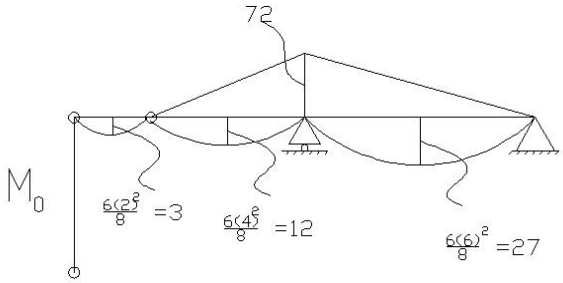
$$\delta_{10} = \frac{2}{3}2(3)\frac{1}{2} - \frac{2}{3}12(4)1 - \frac{2}{3}27(6)1 + \frac{1}{2}4(72)\frac{2}{3}2 + \frac{1}{2}6(72)\frac{2}{3}2$$

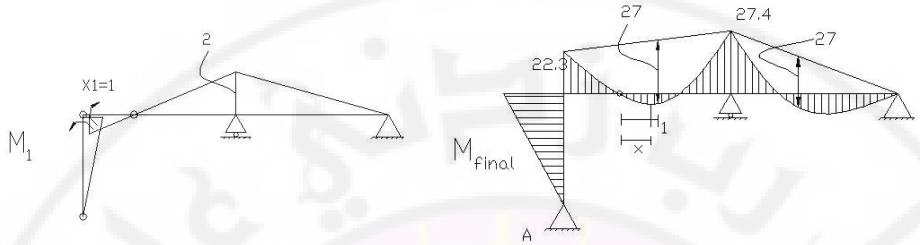
$$= 2 - 32 - 108 + 192 + 288 = 342$$

$$\delta_{11} = \frac{1}{2}4(1)\frac{2}{3} + \frac{1}{2}2\frac{2}{3} + \frac{1}{2}4(2)\frac{2}{3}2 + \frac{1}{2}6(2)\frac{2}{3}2$$

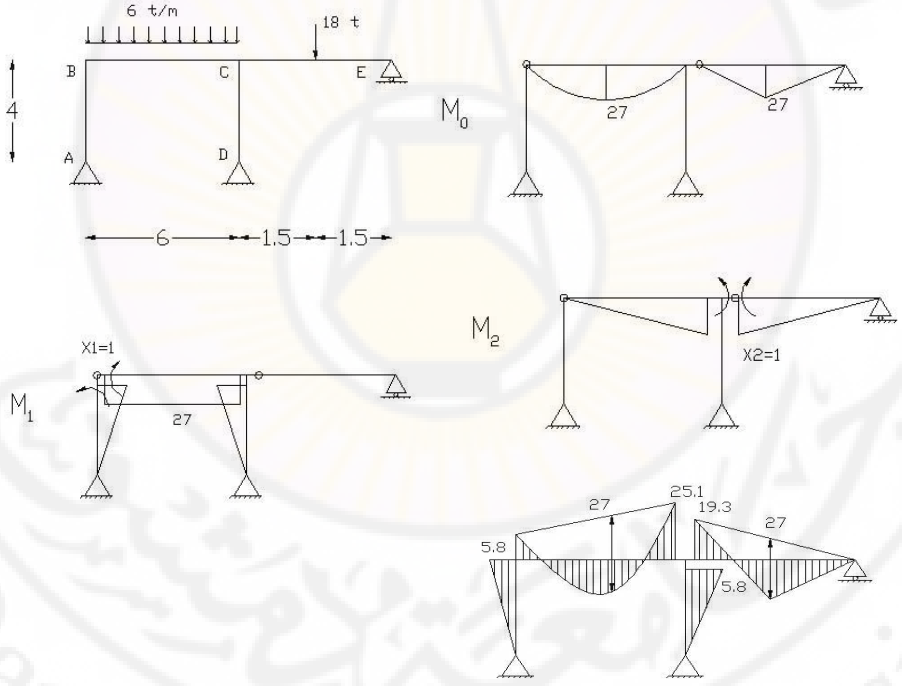
$$= \frac{1}{3}(4+2+16+24) = \frac{46}{3} = 15.333$$

$$X_1 = -\frac{342}{15.333} = -22304$$


72
M_0
$\frac{6(2)^2}{8}=3$
$\frac{6(4)^2}{8}=12$
$\frac{6(6)^2}{8}=27$


M_1
X1=1
2
M_{final}
27
27.4
27
22.3
1
x
A

مسألة (1 – ب) :


6 t/m
18 t
B
C
E
A
D
4
6
1.5
1.5
M_0
27
27
M_1
X1=1
27
M_2
X2=1
27
25.1
19.3
27
5.8
5.8

$$\delta_{10} = \frac{2}{3}6(27) = 108$$

$$\delta_{20} = \frac{2}{3}6(27)\frac{1}{2} + \frac{1}{2}6(27)\frac{1}{2} = 108$$

$$= 3(27)\left(\frac{1}{2} + \frac{1}{3}\right) = 3(37)\frac{7}{6} = 94.5$$

$$\delta_{11} = 1(6) + \frac{1}{2}4(1)\frac{2}{3}2 = \frac{26}{3} = 8.67$$

$$\delta_{12} = \frac{1}{2}6 = 3$$

$$\delta_{22} = \frac{1}{2}6\left(\frac{2}{3}\right)2 = 4$$

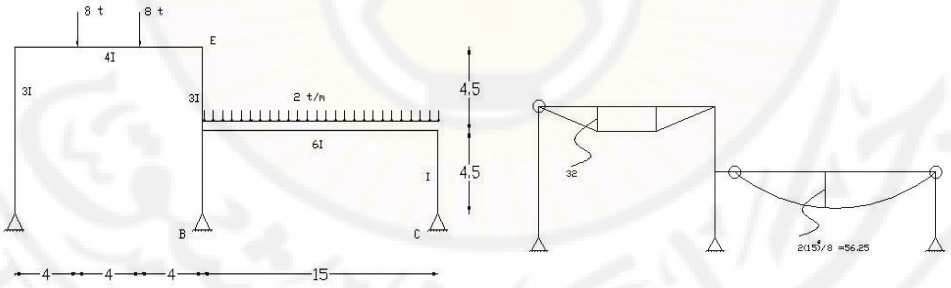
$$\begin{matrix} -\frac{3}{8.67} \\ +1 \end{matrix}\begin{bmatrix} 8.67 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -108 \\ -94.5 \end{bmatrix}$$

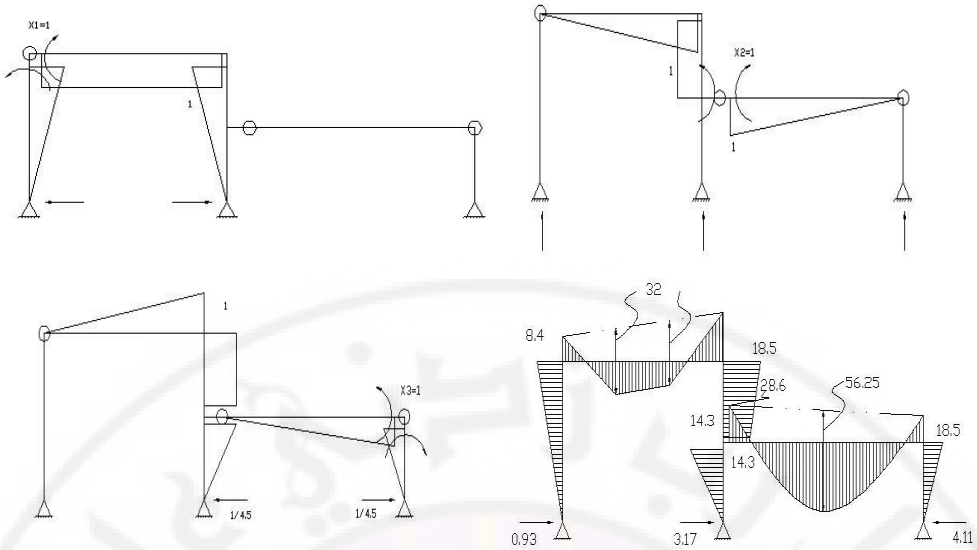
$$2.9615X_2 = -57.1539$$

$$X_2 = -19.3$$

$$X_1 = -5.8$$

مسألة (1 – جـ) :





$$\delta_{10} = 8(32)\frac{1}{4} = 64$$

$$\delta_{20} = 8(32)\frac{1}{4}\frac{1}{2} + \frac{2}{3}(15)(56.25)\frac{1}{2}\frac{1}{6} = 32 + \frac{5}{6}56.25 = 78.875$$

$$\delta_{30} = -32 + \frac{5}{6}56.25 = 14.875$$

$$\delta_{11} = \frac{1}{2}9\frac{2}{3}\frac{1}{3}2 + 1(12)\frac{1}{4} = 5$$

$$\delta_{12} = \frac{1}{2}12\frac{1}{4} + \frac{1.5}{2}\frac{1}{3} = 1.5 + \frac{1.5^2}{2} = 2.625$$

$$\delta_{22} = \frac{1}{2}12\frac{1}{4} + 4.5 + \frac{1}{2}15\frac{2}{3}\frac{1}{6} = 1 + 1.5 + \frac{5}{6} = 3.333$$

$$\delta_{23} = -1 - 1.5 + \frac{1}{2}15\frac{1}{3}\frac{1}{6} = -2.083$$

$$\delta_{33} = 1 + 1.5 + \frac{1}{2}4.5\frac{2}{3}\frac{1}{3} + \frac{5}{6} + \frac{1}{2}4.5\frac{2}{3} = 5.333$$

$$\begin{matrix} \\ -\frac{2.62}{5} \\ +1 \\ \hline \frac{1}{=} \end{matrix}\begin{bmatrix} 5 & 2.625 & -2.875 \\ 2.625 & 3.333 & -2.083 \\ -2.875 & -2.083 & 5.333 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -64 \\ -78.875 \\ -14.875 \end{bmatrix}$$

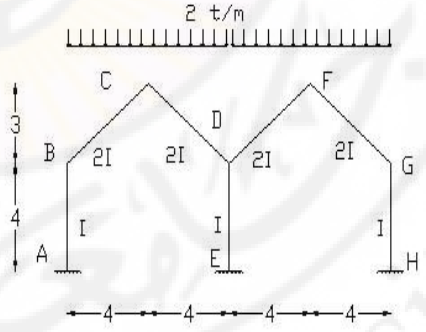
$$\begin{matrix} \\ \frac{0.5736}{1.955} \\ 1 \\ \hline \end{matrix}\begin{bmatrix} 5 & 2.625 & -2.875 \\ 0 & 1.955 & -0.5736 \\ 0 & -0.5736 & 3.680 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -64 \\ -45.275 \\ -51.675 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2.625 & -2.875 \\ 0 & 1.955 & -0.5736 \\ 0 & 0 & 3.512 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -64 \\ -45.275 \\ -64.959 \end{bmatrix}$$

$$X_3 = -18.5$$
$$X_2 = -28.6$$
$$X_1 = -8.4$$

مسألة (1 – د) :

$$\delta_{10} = \frac{2}{3}5(4)\frac{1}{2}\frac{1}{2} + \frac{1}{2}4\frac{64}{3}\left(\frac{1}{3} + \frac{2}{3}\frac{10}{6}\right)$$
$$= \frac{10}{3} + \frac{128}{3}\frac{13}{9} = 64.963$$
$$\delta_{20} = 2\frac{2}{3}5(4)\frac{1}{2}\frac{1}{2} - \frac{1}{2}4\frac{64}{3}\frac{2}{3}\frac{4}{3}$$
$$= \frac{20}{3} - \frac{1024}{27} = -31.259$$

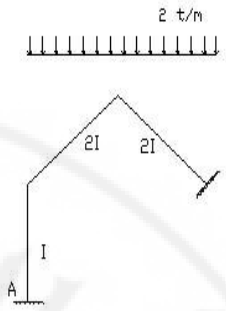


$$\delta_{30} = -\frac{2}{3}5(4)\frac{1}{2}\frac{1}{2} + \frac{1}{2}4\frac{64}{3}\frac{2}{3}\frac{2}{3}$$

$$= \frac{10}{3} + \frac{512}{27} = 22.296$$

$$\delta_{11} = \frac{1}{2}5\frac{2}{3}\frac{1}{2} + \frac{4}{6}\left[2 + 2\frac{100}{6} + 2\frac{10}{6}\right]$$

$$= \frac{5}{6} + 7.259259 = 8.093$$



$$\delta_{12} = \frac{1}{2}5\frac{1}{3}\frac{1}{2} - \frac{1}{2}4\frac{4}{3}\left(\frac{1}{3} + \frac{2}{3}\frac{10}{6}\right) = -3.43$$

$$\delta_{13} = \frac{1}{2}4\frac{2}{3}\left(\frac{1}{3} + \frac{2}{3}\frac{10}{6}\right) = 1.926$$

$$\delta_{22} = \frac{1}{2}5\frac{2}{3}2\frac{1}{2} + \frac{1}{2}4\frac{4}{3}\frac{2}{3}\frac{4}{3} = 4.037$$

$$\delta_{23} = \frac{1}{2}5\frac{1}{3}\frac{1}{2} - \frac{1}{2}4\frac{4}{3}\frac{2}{3}\frac{2}{3} = -0.7685$$

$$\delta_{33} = \frac{1}{2}5\frac{2}{3}\frac{1}{2} + \frac{1}{2}4\frac{2}{3}\frac{2}{3}\frac{2}{3} = 1.426$$

$$\begin{bmatrix} 8.093 & -3.435 & 1.926 \\ -3.435 & 4.037 & 0.7685 \\ 1.926 & -0.7685 & 1.426 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -64.963 \\ +31.259 \\ -22.293 \end{bmatrix}$$

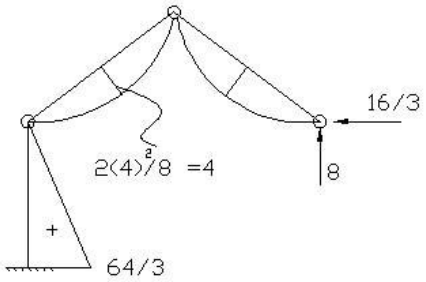
$$\begin{bmatrix} 8.093 & -3.435 & 1.926 \\ 0 & 2.579 & 0.049 \\ 0 & 0.049 & 0.968 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -64.963 \\ +3.686 \\ +6.833 \end{bmatrix}$$

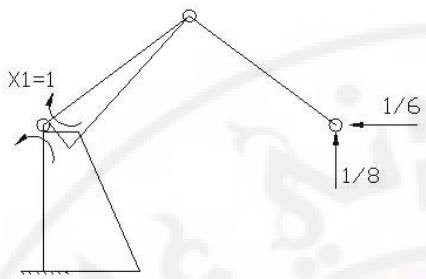
$$0.967X_3 = -6.903$$

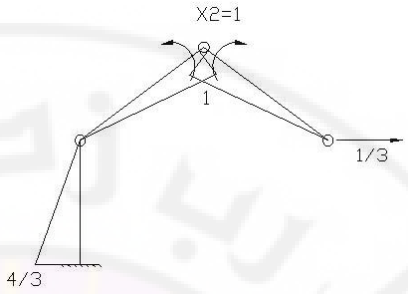
$$X_3 = -7.14$$

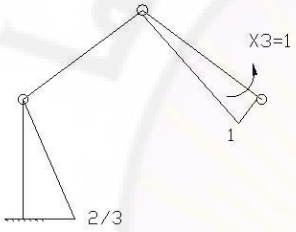
$$X_2 = -1.565$$

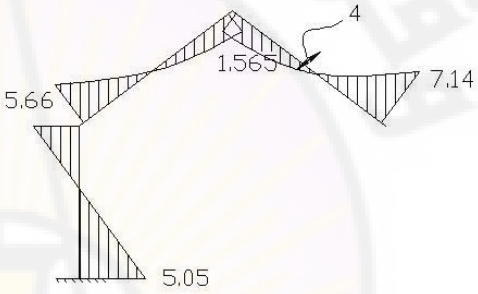
$$X_1 = -5.66$$


16/3
2(4)²/8 =4
8
+
64/3

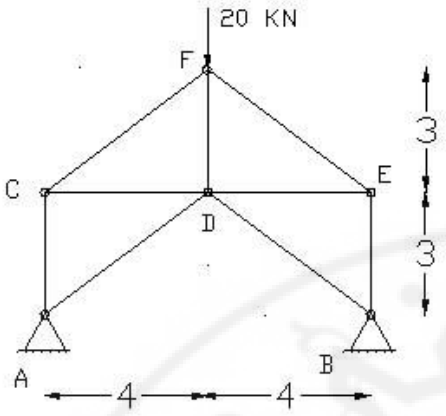

X1=1
1/6
1/8

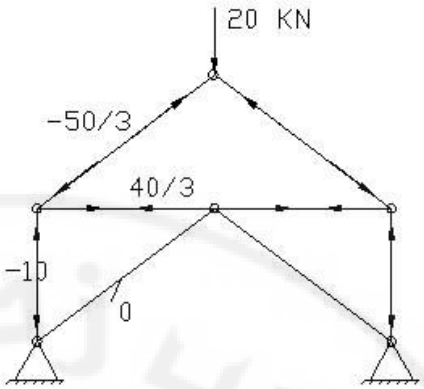

X2=1
1
1/3
4/3

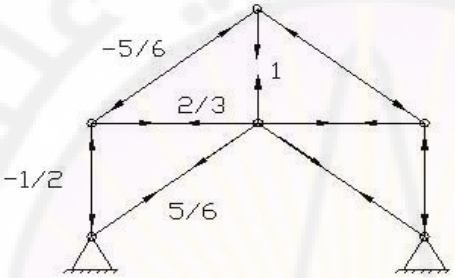

X3=1
1
2/3


4
1.565
5.66
7.14
5.05

مسألة (2 – آ) :







القضيب	N_0	N_1	L	N_{Final}
AC	-10	-1/2	3	-4.53
AD	0	+5/6	5	-9.12
BD	0	+5/6	5	-9.12
BE	-10	-1/2	3	-4.53
CD	40/3	2/3	4	6.04
CF	-50/3	-5/6	5	-7.55
DE	40/3	2/3	4	6.04
DF	0	1	3	-10.94
EF	-50/6	-5/6	5	-7.55

$$\delta_{10} = \sum N_1 \frac{N_0}{EA} L$$

$$= 2\left[(-10)\left(-\frac{1}{2}\right)3 + \left(\frac{40}{3}\right)\left(\frac{2}{3}\right)4 + \left(-\frac{50}{3}\right)\left(-\frac{5}{6}\right)5\right]$$

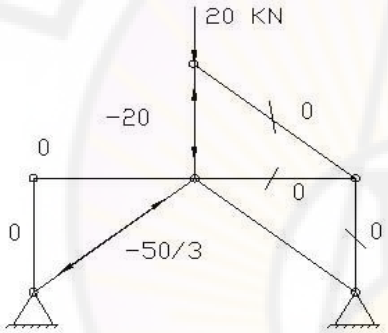
$$= 2\left(12 + \frac{320}{9} + \frac{1250}{18}\right) = 240$$

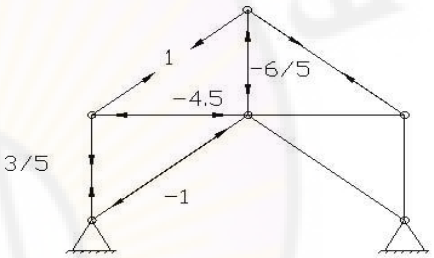
$$\delta_{11} = 2\left[\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)3 + \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)5(2) + \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)4\right] + 3$$

$$= 2\left(\frac{3}{4} + \frac{125}{18} + \frac{16}{9}\right) + 3 = 21.944$$

$$X_1 = -\frac{240}{21.944} = -10.94t$$

مسألة (2 – آ) أسلوب ثاني :





تناظر	القضيب	N_0	N_1	L	N_{Final}
2	*AC*	0	3/5	3	-4.53
2	*AD*	-50/3	-1	5	-9.12
2	*CD*	0	-4/5	4	+6.04
2	*CF*	0	+1	5	-7.55
1	*DF*	-20	-6/5	3	-10.94

$$\delta_{10} = 2\left[\left(-\frac{50}{3}\right)(-1)5\right] + (-20)\left(-\frac{6}{5}\right)3$$

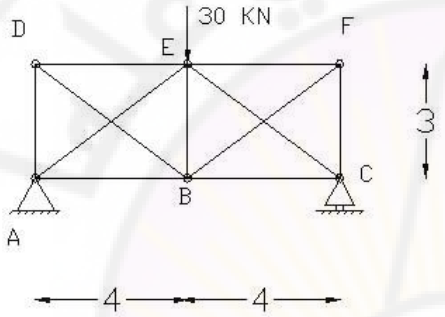
$$= \frac{500}{3} + \frac{360}{5} = 238.667$$

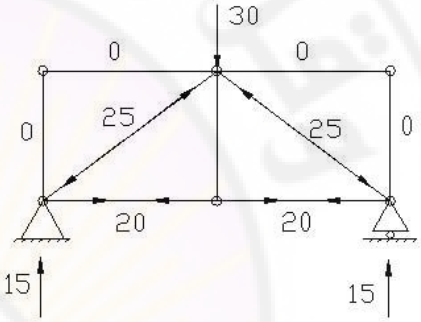
$$\delta_{11} = 2\left[\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)3 + 5(2) + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\right] + \left(\frac{6}{5}\right)\left(\frac{6}{5}\right)3$$

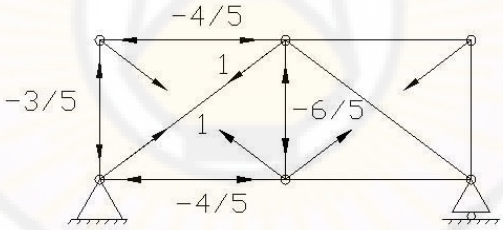
$$= \frac{2}{25}(27 + 250 + 64 + 54) = 31.6$$

$$X_1 = -7.55$$

مسألة (2 – ب) :







تناظر	القضيب	N_0	N_1	L	N_{Final}
2	AB	20	-4/5	4	+11.8
	AD	0	-3/5	3	-6.2
	AE	-25	1	5	-14.7
	BD	0	1	5	10.3
	DE	0	-4/5	4	-8.2
	BE	0	-6/5	3	-12.3

$$\delta_{10} = 2(20)\left[\left(-\frac{4}{5}\right)4 + (-25)1(5)\right]$$

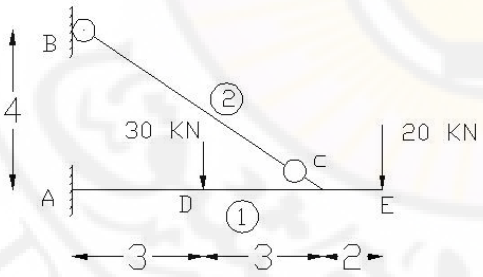
$$= 2(-64-125) = -378$$

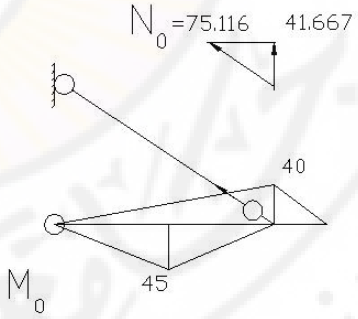
$$\delta_{11} = 2\left[\left(-\frac{4}{5}\right)\left(-\frac{4}{5}\right)4(2) + \left(-\frac{3}{5}\right)\left(-\frac{3}{5}\right)3 + 5(2)\right]$$

$$= \frac{2}{25}(128-27+250+54) = 36.72$$

$$X_1 = \frac{378}{36.72} = 10.3$$

مسألة (3) :





$$\delta_{10} = \frac{1}{E_1 I_1}\left[\left(\frac{1}{2}\right)6(40)\frac{1}{3} - \frac{1}{2}6(45)\frac{1}{2}\right] - \frac{1}{E_2 A_2}\left[75.116(0.3005)\sqrt{52}\right]$$

$$2.1\times10^4\,\delta_{10} = \frac{1}{3}[40 - 1.5(45)] - \frac{1}{4}[162.77]$$

$$= -49.86$$

$$2.1\times10^4\,\delta_{11} = \frac{1}{3}\left[\frac{1}{2}6\frac{2}{3}\right] + \frac{1}{4}\left[0.3005^2\sqrt{52}\right]$$

$$= 0.829$$

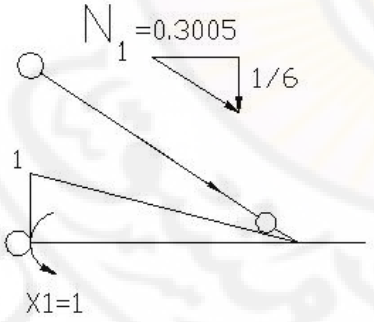
$$X_1 = M_A = \frac{49.86}{0.8295} = 60.11$$

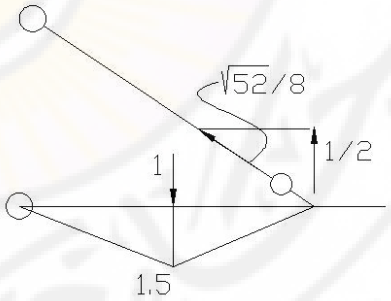
$$M_D = -5.55$$

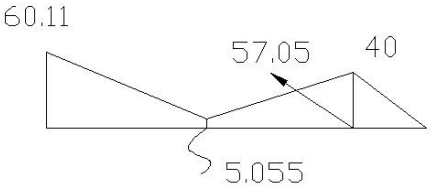
$$2.1\times10^4\,\delta_D = \frac{1}{3}\left[2\frac{1}{2}3(45) - \frac{1}{2}6(1.5)\frac{60.11+40}{2}\right] + \frac{1}{4}\left[57.05(0.9014)\sqrt{52}\right]$$

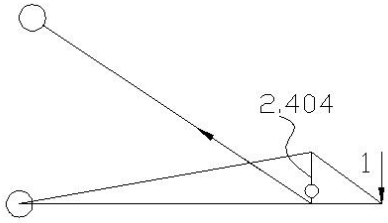
$$= -30.08 + 92.71$$

$$\delta_D = 2.982mm = 3.0mm$$









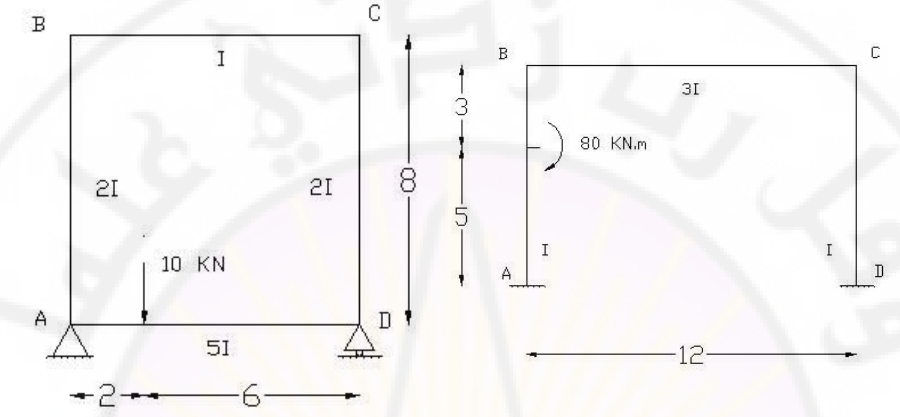
$$2.1\times10^4\delta_E=\frac{1}{3}\left[\frac{1}{2}6(45)+\frac{1}{2}6(2)\left(\frac{2}{3}40+\frac{1}{3}60.11\right)\right]+\frac{1}{4}\left[57.05(2.404)\sqrt{52}\right]+\frac{1}{2}2(40)\frac{2}{3}2$$

$$=66.18+247.25$$

$$\delta_E=14.9$$

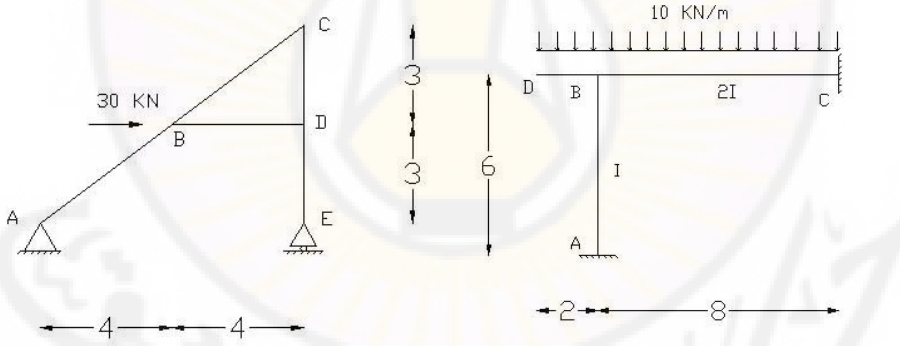
المركز المرن والتشبيه العمودي

1- يطلب حل المنشآت الآتية بطريقة المركز المرن والتشبيه العمودي :



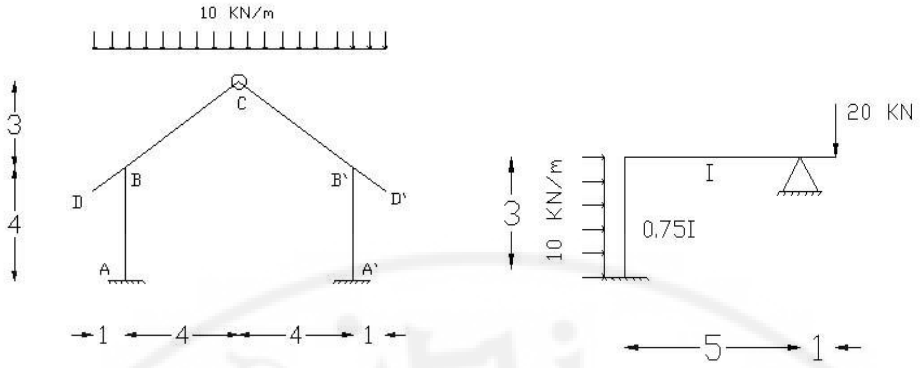
- ب -

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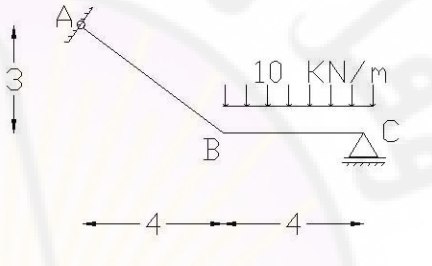
- جـ -

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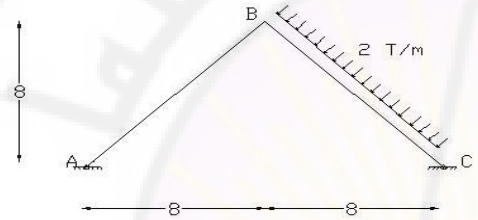


- د -

- هـ -



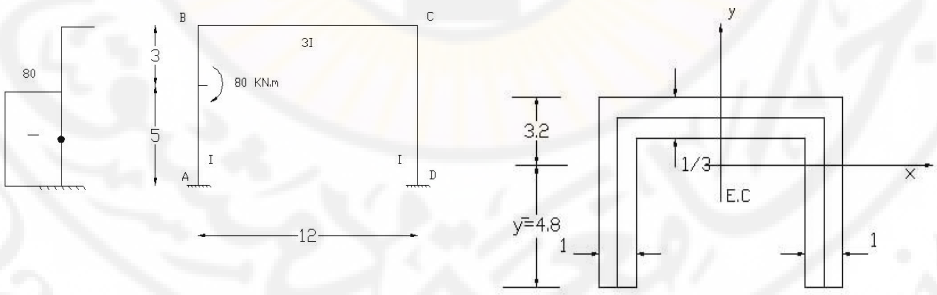
- و -



- ز -

2- استخدم خاصية التناظر والتناظر العكسي في حل الإطارين في الأعلى

مسألة (1 – آ) :



$$\bar{y} = \frac{2(8)4 + \frac{1}{3}12(8)}{2(8) + \frac{1}{3}12} = \frac{64+32}{20} = \frac{96}{20} = 4.8$$

$$I_x = 2\frac{8^3}{3} - 20(3.2)^2 = 136.53$$

$$I_y = \frac{1}{2}\frac{12^3}{12} + 2(8)6^2 = 624$$

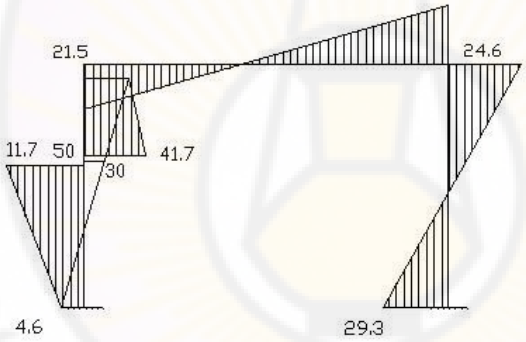
$$N = -5(80) = -400$$

$$M_x = +400(2.3) = 920$$

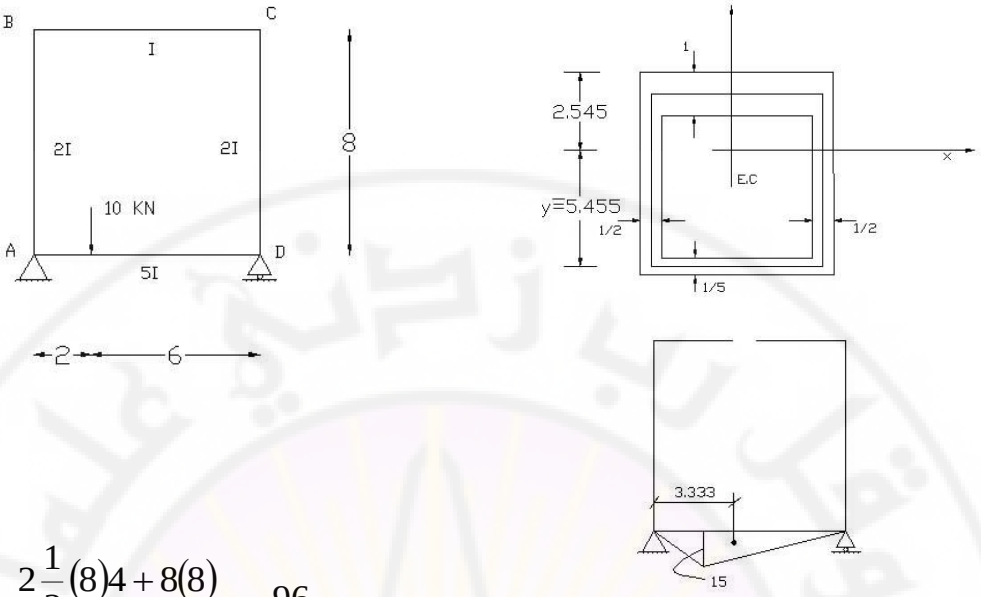
$$M_y = +400(6) = 2400$$

$$M_{Final} = M_0 - \frac{920}{136.53}y - \frac{2400}{624}x + \frac{400}{20}$$

$$= M_0 - 6.738y - 3.846x + 20$$



مسألة (1 – ب) :



$$\bar{y} = \frac{2\frac{1}{2}(8)4 + 8(8)}{2\frac{1}{2}8 + 8 + \frac{1}{5}8} = \frac{96}{17.6} = 5.455$$

$$I_x = 2\frac{1}{2}\frac{8^3}{3} + 2\frac{1}{2}8(1.455)^2 + 8(2.545)^2 + \frac{1}{5}5(5.455)^2 = 159.03$$

$$I_y = \left(1+\frac{1}{5}\right)\frac{8^3}{12} + 2\frac{1}{2}8(4)^2 = 179.2$$

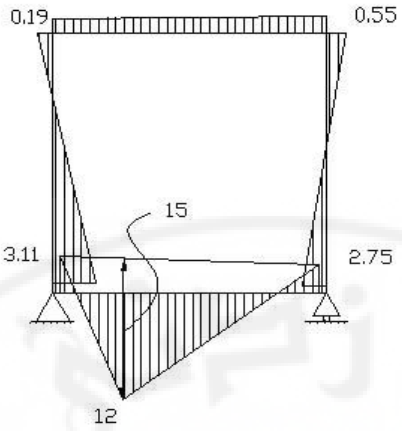
$$N = -\frac{1}{2}8(15) - = -400$$

$$M_x = +12(5.455) = 65.46$$

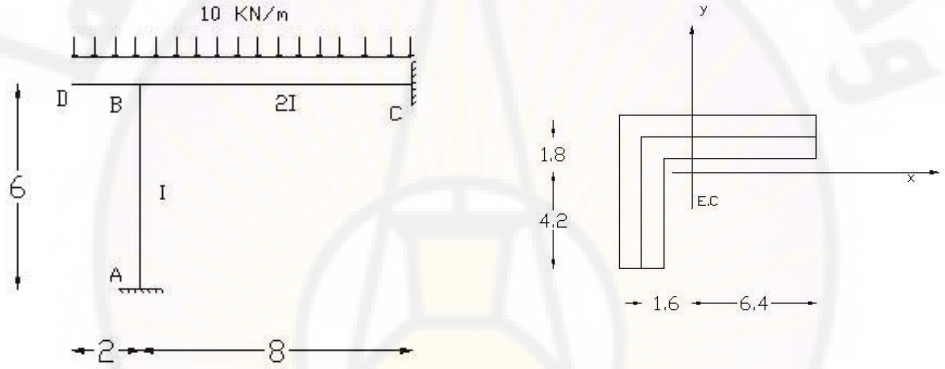
$$M_y = 12(0.667) = 8$$

$$M_{Final} = M_0 - \frac{65.46}{159.03}y - \frac{8}{179.2}x + \frac{12}{17.6}$$

$$= M_0 - 6.738y - 3.846x + 20$$



مسألة (1 – جـ) :



$$\bar{x} = \frac{\frac{1}{2}8(4)}{1(6)+\frac{1}{2}8} = \frac{16}{10} = 1.6$$

$$\bar{y} = \frac{1(6)3}{10} = 1.8$$

$$I_x = \frac{6^3}{3} - 10(1.8)^2 = 39.6$$

$$I_y = \frac{1}{2}\frac{8^3}{3} - 10(1.6)^2 = 59.73$$

$$I_{xy} = \frac{1}{2}8(2.4)(1.8) + 6(1.6)(1.2) = 28.8$$

$$I'_x = 39.6\left[1 - \frac{28.8^2}{39.6(59.73)}\right] = 25.71$$

$$I'_y = 38.78$$

$$N = \frac{1}{2}8(20)\frac{1}{2} + \frac{2}{3}8(80)\frac{1}{2} = +173.33$$

$$M_x = +173.338(1.8) = 312$$

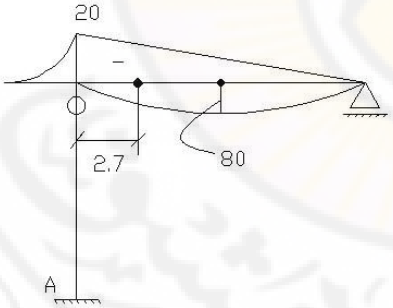
$$M_y = -40(1.07) + 213.33(2.4) = 469.33$$

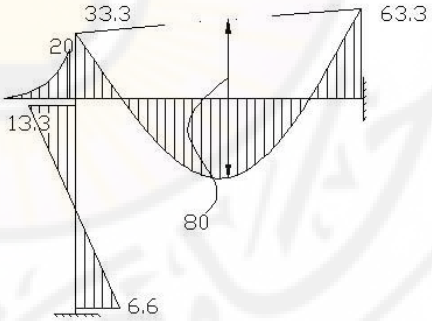
$$M'_x = 312 - 469.33\frac{28.8}{59.73} = 85.7$$

$$M'_y = 469.33 - 312\frac{28.8}{39.6} = 242.4$$

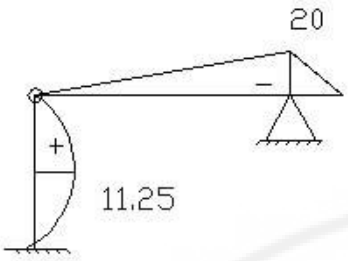
$$M_{Final} = M_0 - \frac{85.7}{25.71}y - \frac{242.4}{38.78}x - \frac{173.3}{10}$$

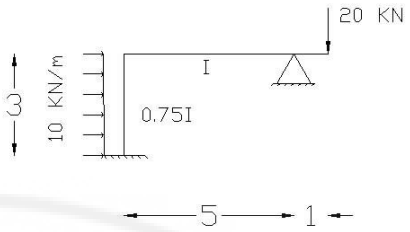
$$= M_0 - 3.33y - 6.25x - 17.33$$

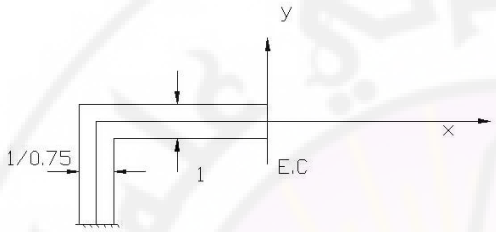


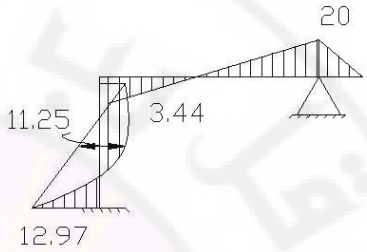


مسألة (1 – د) :









$$I_x = \frac{1}{0.75}\frac{3^3}{3} = 12$$

$$I_y = \frac{5^3}{3} + \frac{1}{0.75}3(5)^2 = 141.67$$

$$I_{xy} = \frac{1}{0.75}3(5)(1.5) = 30$$

$$I'_x = 12\left[1 - \frac{30^2}{12(141.67)}\right] = 5.647$$

$$I'_y = 66.67$$

$$M_x = -\frac{2}{3}3(11.25)\frac{1}{0.75}(1.5) = -45$$

$$M_y = +\frac{1}{2}5(20)\frac{5}{3} - \frac{2}{3}3(11.25)\frac{1}{0.75}5 = -66.67$$

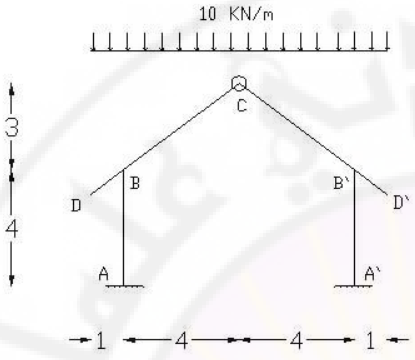
$$M'_x = -45 + 66.67\frac{30}{141.67} = -30.88$$

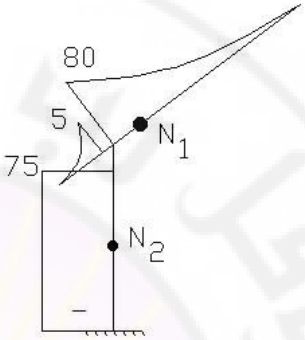
$$M'_y = -66.67 + 45\frac{30}{12} = 45.83$$

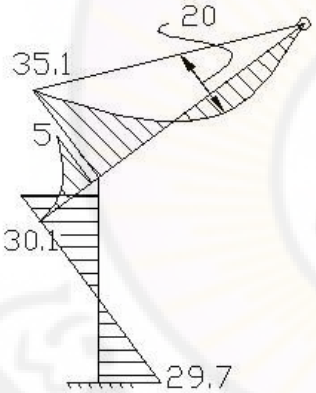
$$M_{Final} = M_0 + \frac{30.88}{5.647}y - \frac{45.83}{66.67}x$$

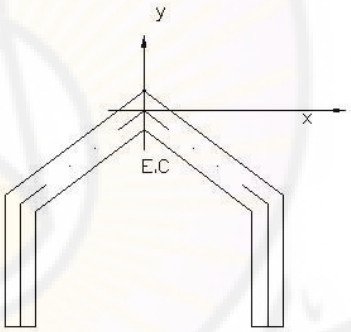
$$= M_0 + 5.468y - 0.687x$$

مسألة (1 – هـ) :









$$I_x = \left[5\frac{3^3}{3} + \frac{4^3}{12} + 4(5)^2 \right] 2 = 240.67$$

$$I_y = \left[5\frac{4^2}{3} + 4(4)^2 \right] 2 = 181.33$$

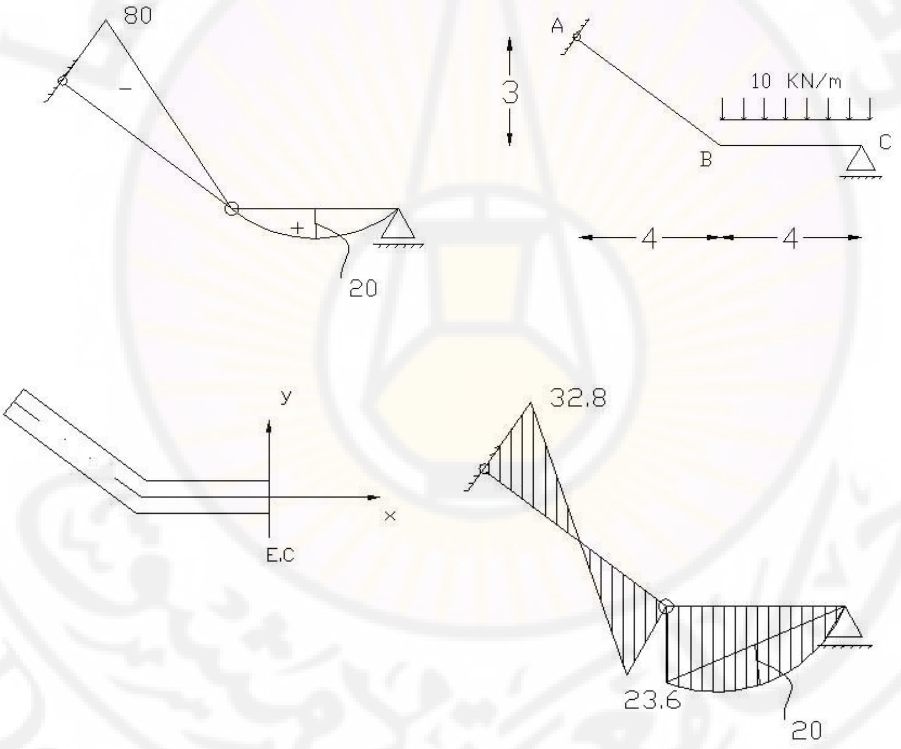
$$M_x = \left[+\frac{1}{3} 5(80) \frac{3}{4} 3 + 4(75)5 \right] = 3600$$

$$M_y = 0$$

$$M_{Final} = M_0 - \frac{3600}{240.67} y$$

$$= M_0 - 14.96y$$

مسألة (1 – و) :



$$I_x = \infty$$

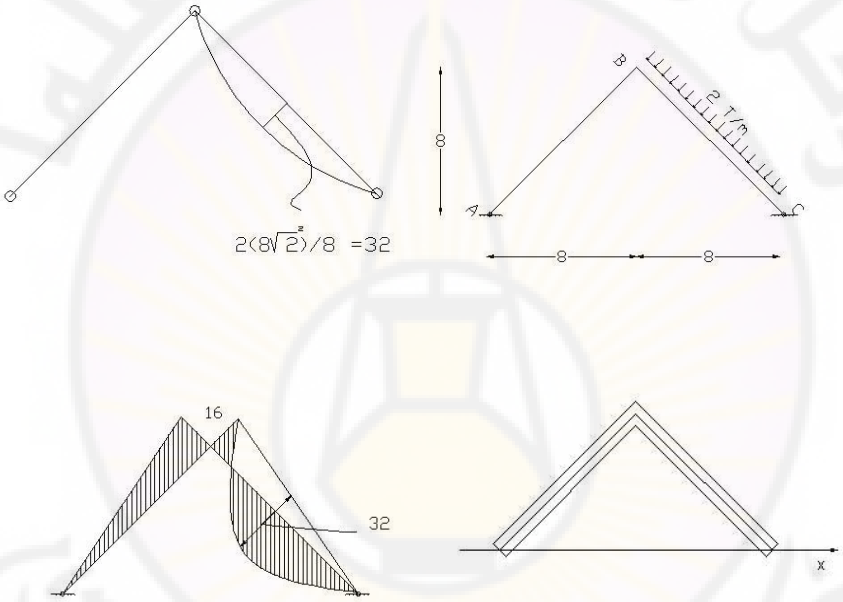
$$I_y = \frac{4^3}{3} + 5\frac{4^2}{12} + 5(6)^2 = 208$$

$$M_y = -\frac{2}{3}4(20)2 + \frac{1}{2}5(80)(6.667) = 1226.7$$

$$M_{Final} = M_0 - \frac{1226.7}{208}x$$

$$= M_0 - 5.897x$$

مسألة (1 – ز) :



$$I_x = 28\sqrt{2}\,\frac{8^2}{3} = 482.7$$

$$M_x = \frac{2}{3}8\sqrt{2}\,4(32)4 = 965.4$$

$$M_{Final} = M_0 - \frac{965.4}{482.7}\,y$$

$$= M_0 - 2y$$

مسألة (1 – حـ) :

$$\bar{x} = \frac{4(2)+5(2)}{3+4+5} = 1.5$$

$$\bar{y} = \frac{3(1.5)+5(1.5)}{12} = 1$$

$$I_x = \frac{3^3}{3} + \frac{5(3)^2}{3} - 12(1)^2 = 12$$

$$I_y = \frac{4^3}{3} + \frac{5(4)^2}{3} - 12(1.5)^2 = 21$$

$$I_{xy} = 5\frac{3(4)}{12} + 5(0.5)(-0.5) + 3(0.5)(1.5) + 4(-0.5)(-1) = 8$$

$$I'_x = 12\left[1 - \frac{8^2}{12(21)}\right] = 8.952$$

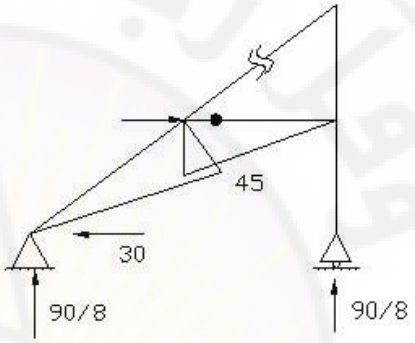
$$I'_y = 15.667$$

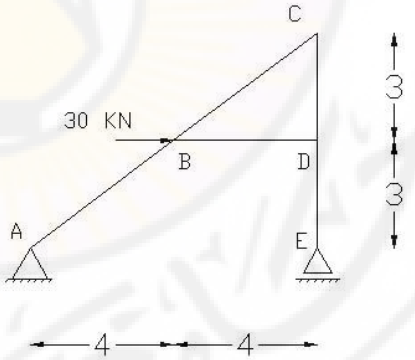
$$N = \frac{1}{2}4(45) = -90$$

$$M_x = +90$$

$$M_y = +90(1.167) = 105$$

$$M'_x = 90 - 105\frac{8}{21} = 50$$

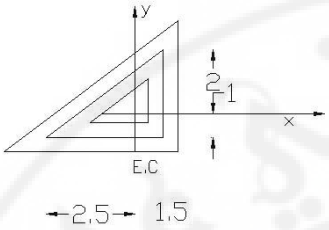


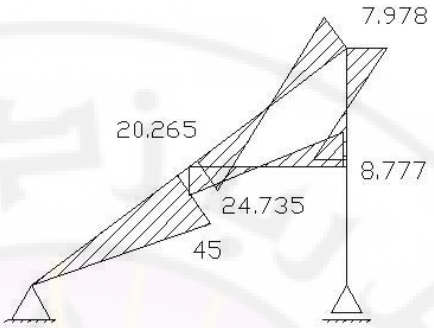


$$M'_y = 105 - 90\frac{8}{12} = 45$$

$$M_{Final} = M_0 - \frac{50}{8.952}y - \frac{45}{15.667}x + \frac{90}{12}$$

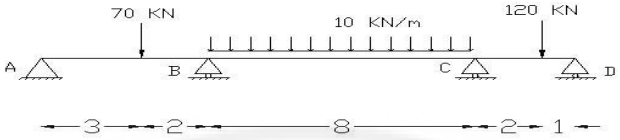
$$= M_0 - 5.585y - 2.872x + 7.5$$



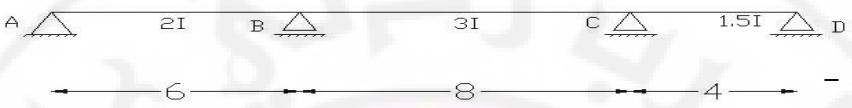


مسائل على طريقة العزوم الثلاثة

- يطلب حل المنشآت الآتية بطريقة العزوم الثلاثة



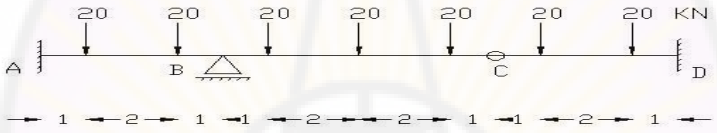
- آ -



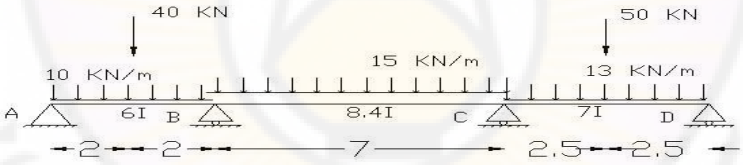
- ب -

$\Delta_B = 1cm$ $\Delta_C = 0.5\ cm$

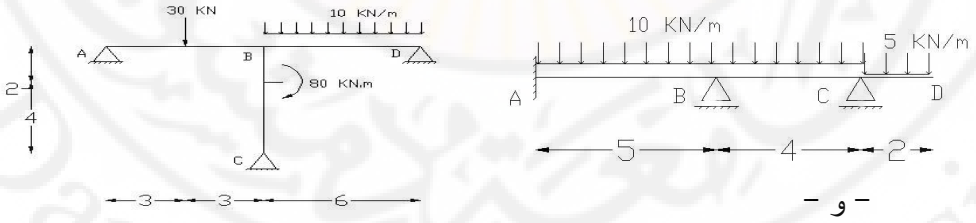
$E = 3x10^7\ KN/m^2$ $I = 0.01\ cm^2$



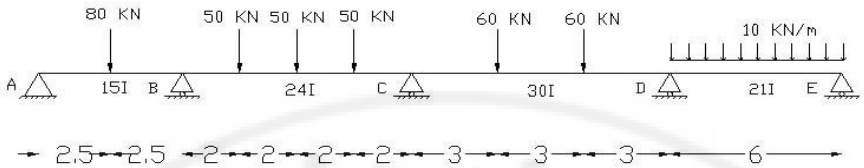
- جـ -



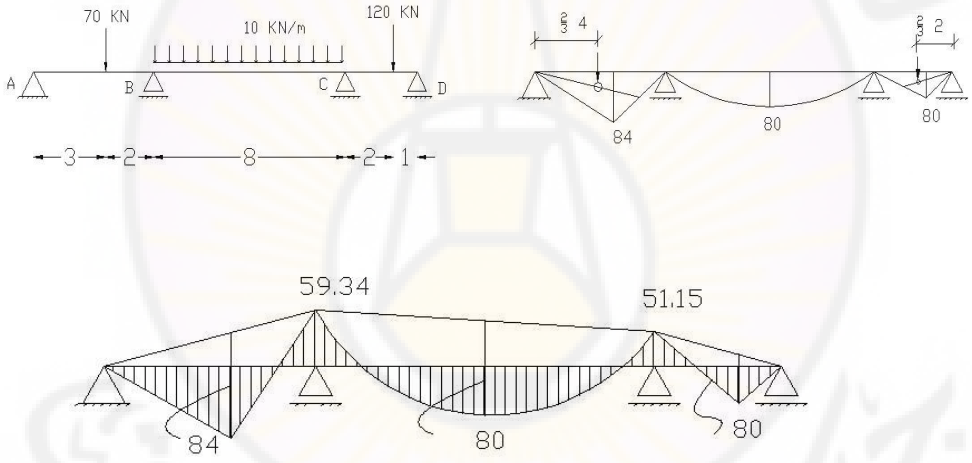
- هـ -



- و -



مسألة (آ) :



$$2M_B(5+8)+M_C(8)=-6\left[\frac{1}{2}5(84)\right]\frac{\frac{2}{3}4}{5}-6\left[\frac{2}{3}8(80)\right]\frac{1}{2}$$

(1) $26M_B + 8M_C = -1952$

$$M_B(8) + 2M_C(8+3) = -6\left[\frac{2}{3}8(80)\right]\frac{1}{2} - 6\left[\frac{1}{2}3(80)\right]\frac{\frac{2}{3}2}{3}$$

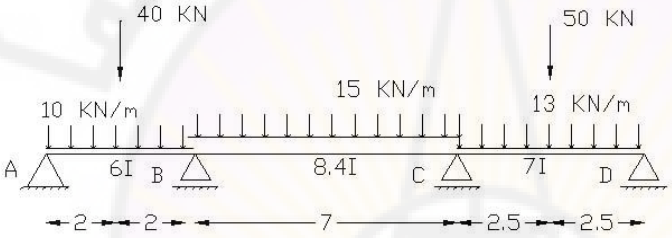
(2) $8M_B + 22M_C = -1600$

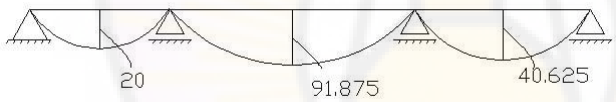
$$-\frac{8}{26}\begin{bmatrix}26 & 8\\ 8 & 22\end{bmatrix}\begin{bmatrix}M_B\\ M_C\end{bmatrix} = \begin{bmatrix}-1952\\ -1600\end{bmatrix}$$

$$M_C = -51.15$$

$$M_B = -59.34$$

مسألة (ب) :







40 62.5

$$2M_B\left(\frac{4}{8}+\frac{7}{8.4}\right) + M_C\left(\frac{7}{8.4}\right) = -6\left[\frac{2}{3}4(20)\right]\frac{1}{6}\frac{1}{2} - 6\left[\frac{1}{2}4(40)\right]\frac{1}{6}\frac{1}{2} - 6\left[\frac{2}{3}7(91.875)\right]\frac{1}{8.4}\frac{1}{2}$$

(1) $3M_B + 0.833M_C = -219.792$

$$2M_B\left(\frac{7}{8.4}\right) + 2M_C\left(\frac{7}{8.4}+\frac{5}{7}\right) = -6\left[\frac{2}{3}7(91.875)\right]\frac{1}{8.4}\frac{1}{2} - 6\left[\frac{2}{3}5(40.625)\right]\frac{1}{7}\frac{1}{2} - 6\left[\frac{1}{2}5(62.5)\right]\frac{1}{7}\frac{1}{2}$$

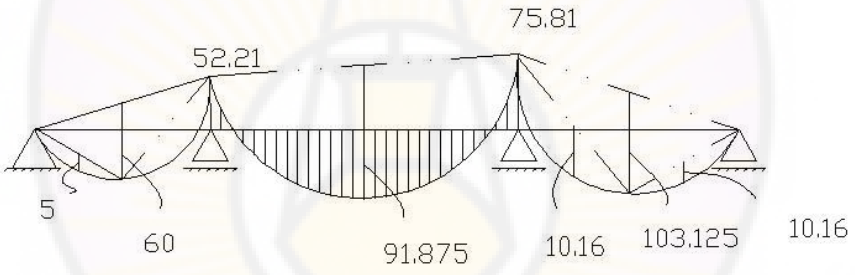
(2) $0.833M_B + 3.95M_C = -278.12$

$$-\frac{0.833}{3} \quad \begin{bmatrix} 3 & 0.833 \\ 0.833 & 3.095 \end{bmatrix}\begin{bmatrix} M_B \\ M_C \end{bmatrix} = \begin{bmatrix} -219.792 \\ -278.125 \end{bmatrix}$$

$$1$$

$M_C = -75.81$

$M_B = -52.21$



مسألة (جـ) :

$$2M_A(5)+M_B(5)=-6\left[\frac{2}{3}5(31.25)\right]\frac{1}{2}$$

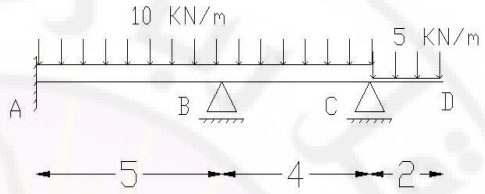
(1) $\quad 2M_A+M_B=-62.5$

$$M_A(5)+2M_B(4+5)=-6\left[\frac{2}{3}5(31.25)\right]\frac{1}{2}-6\left[\frac{2}{3}4(20)\right]\frac{1}{2}+6\left[\frac{1}{2}4(10)\right]\frac{1}{3}$$

(2) $\quad 5M_A+18M_B=-432.5$

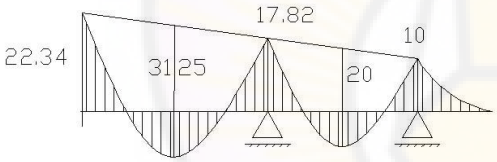
$$-\frac{5}{2}\begin{bmatrix}2 & 1\\ 5 & 18\end{bmatrix}\begin{bmatrix}M_A\\ M_B\end{bmatrix}=\begin{bmatrix}-62.5\\ -432.5\end{bmatrix}$$

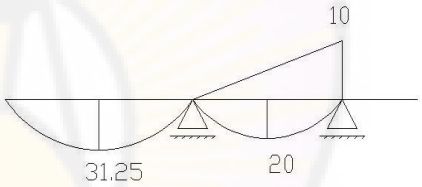
$$\underline{1}$$



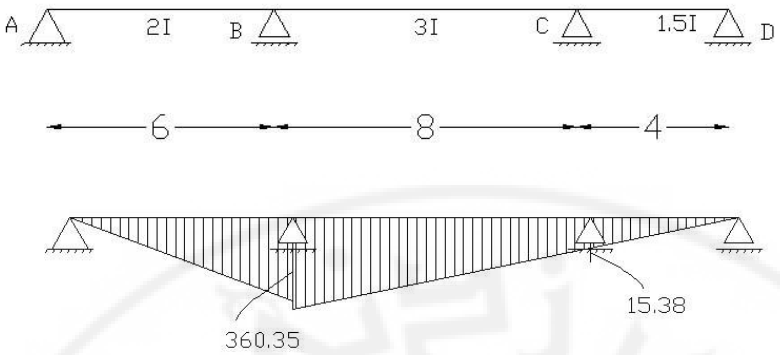
$$M_B=-17.82$$

$$M_A=-22.34$$





مسألة (د) :



$$2M_B\left(\frac{6}{2}+\frac{8}{3}\right)+M_C\left(\frac{8}{3}\right)=+6EI\frac{1\times10^{-2}}{6}+6EI\frac{0.5\times10^{-2}}{8}$$

(1) $$11.33M_B+2.667M_C=4125$$

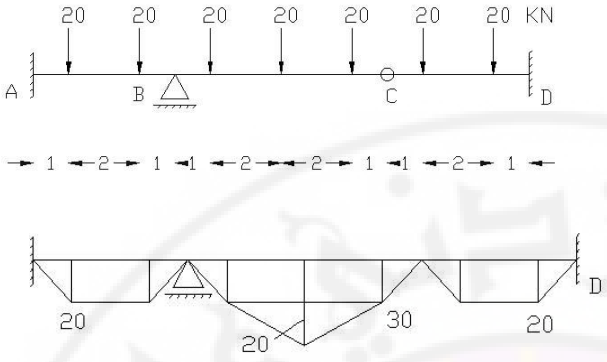
$$2M_B\left(\frac{8}{3}\right)+M_C\left(\frac{8}{3}+\frac{4}{1.5}\right)=-6EI\frac{0.5\times10^{-2}}{8}+6EI\frac{0.5\times10^{-2}}{4}$$

(2) $$2.667M_B+10.67M_C=+1125$$

$$\begin{matrix}-\frac{2.667}{11.33}\\ \hline 1\end{matrix}\begin{bmatrix}11.33 & 2.667\\ 2.667 & 10.67\end{bmatrix}\begin{bmatrix}M_B\\ M_C\end{bmatrix}=\begin{bmatrix}4125\\ 1125\end{bmatrix}$$

$$M_C=+15.38$$
$$M_B=360.35$$

مسألة (هـ) :



$$1/\quad 2M_A(4)+M_B(4)=-6[3(20)]\frac{1}{2}$$

$$(1)\qquad 2M_A+M_B=-45$$

$$2/\quad M_A(4)+2M_B(4+6)=-6[3(20)]\frac{1}{2}-6\left[3(20)+\frac{1}{2}4(20)\right]\frac{1}{2}-6EI\frac{y}{6}$$

$$(2)\qquad 4M_A+20M_B=-750-EIy$$

$$3/\quad 2M_D(4)=-6[3(20)]\frac{1}{2}-6EI\frac{y}{4}$$

$$(3)\qquad 8M_D=-180-1.5EIy$$

$$4/\quad M_B=6Q-20(1+3+5)=6Q-180\ \Big|\ \frac{2}{}$$

$$M_D=-4Q-20(1+3)=-4Q-80\ \Big|\ \frac{3}{}$$

$$(4)\qquad 2M_B+3M_D=-600$$

$$1.5(2)-(3) \quad \Rightarrow \quad 6M_A+30M_B-8M_D=-945 \qquad (*)$$

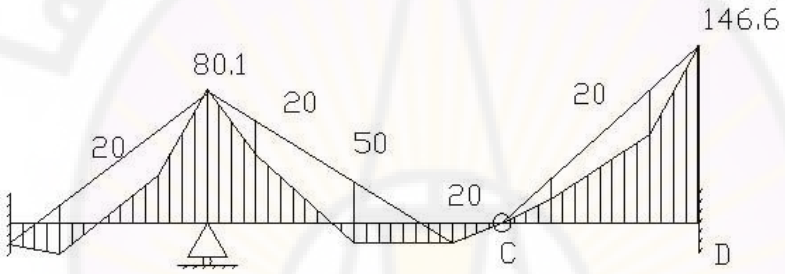
$$(1)\ ,\ (*) \quad \Rightarrow \quad 3(-45-M_B)+30M_B-8M_D=-945$$

$$(**) \qquad 27M_B-8M_D=-810$$

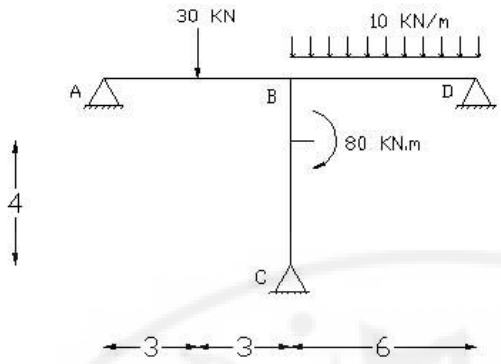
$$\begin{matrix} -\frac{27}{2} \\ 1 \end{matrix} \begin{bmatrix} 2 & 3 \\ 27 & -8 \end{bmatrix} \begin{bmatrix} M_B \\ M_D \end{bmatrix} = \begin{bmatrix} -600 \\ -810 \end{bmatrix}$$

$$M_D=-150.31$$

$$M_B=-74.54$$



مسألة (و) :



$$1/\quad 2M_{BA}(6)+2M_{BC}(6)=-6\left[\frac{1}{2}6(45)\right]\frac{1}{2}+6\left[\frac{1}{2}2\left(\frac{1}{3}80\right)\right]\frac{4+\frac{2}{3}}{6}-6\left[\frac{1}{2}4\left(\frac{2}{3}80\right)\right]\frac{\frac{2}{3}4}{6}$$

$$(1)\qquad M_{BA}+M_{BC}=-47.083$$

$$2/\quad M_{BA}(6)+2M_{BD}=-6\left[\frac{1}{2}6(45)\right]\frac{1}{2}-6\left[\frac{2}{3}6(45)\right]\frac{1}{2}$$

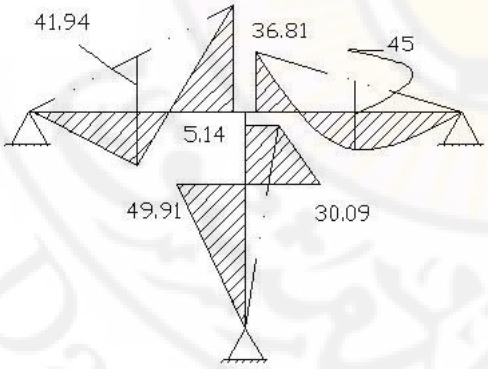
$$(2)\qquad M_{BA}+M_{BD}=-78.75$$

$$(3)\qquad M_{BA}-M_{BC}-M_{BD}=0$$

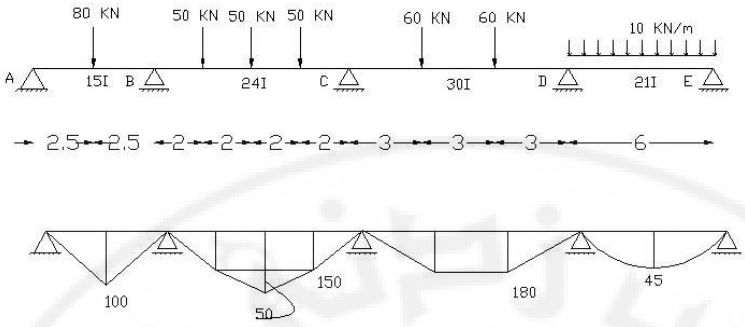
$$(1)+(2)+(3)\quad\Rightarrow\quad 3M_{BA}=-125.83\quad\Rightarrow\quad M_{BA}=41.94$$

$$M_{BC}=-5.14$$

$$M_{BD}=-36.81$$



مسألة (ز) :



$$1/\quad 2M_B\left(\frac{5}{15}+\frac{8}{24}\right)+M_C\left(\frac{8}{24}\right)=-6\left[\frac{1}{2}5(100)\right]\frac{1}{15}\frac{1}{2}-6\left[150(6)+2(50)\right]\frac{1}{24}\frac{1}{2}$$

$$(1)\qquad \frac{4}{3}M_B+\frac{1}{3}M_C=-175$$

$$2/\quad M_B\left(\frac{8}{24}\right)+2M_C\left(\frac{8}{24}+\frac{9}{30}\right)+M_D\left(\frac{9}{30}\right)=-6\left[6(150)+2(50)\right]\frac{1}{24}\frac{1}{2}-6\left[6(180)\right]\frac{1}{30}\frac{1}{2}$$

$$(2)\qquad \frac{1}{3}M_B+\frac{3.8}{3}M_C+\frac{0.9}{3}M_D=-233$$

$$3/\quad M_C\left(\frac{9}{30}\right)+2M_D\left(\frac{9}{30}+\frac{6}{21}\right)=-6\left[6(180)\right]\frac{1}{30}\frac{1}{2}-6\left[\frac{2}{3}6(45)\right]\frac{1}{21}\frac{1}{2}$$

$$(3)\qquad \frac{0.9}{3}M_C+\frac{3.514}{3}M_D=-133.71$$

$$\begin{matrix}-\frac{1}{4}\\ 1\\ \\ \end{matrix}\begin{bmatrix}4 & 1 & 0\\ 1 & 3.8 & 0.9\\ 0 & 0.9 & 3.514\end{bmatrix}\begin{bmatrix}M_B\\ M_C\\ M_D\end{bmatrix}=\begin{bmatrix}-525\\ -699\\ -401.14\end{bmatrix}$$

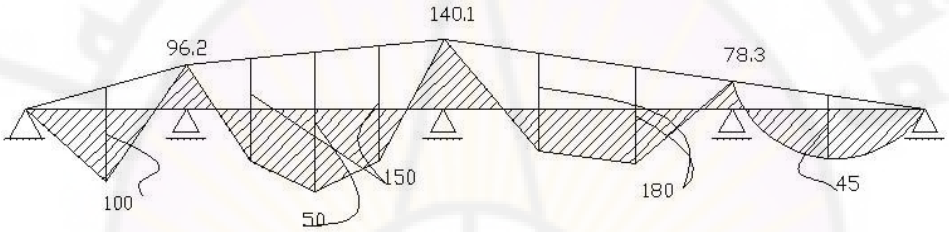
$$\begin{matrix} -\frac{0.9}{3.55} \\ 1 \\ \\ \hline \end{matrix}\begin{bmatrix} 4 & 1 & 0 \\ 0 & 3.55 & 0.9 \\ 0 & 0.9 & 3.514 \end{bmatrix}\begin{bmatrix} M_B \\ M_C \\ M_D \end{bmatrix} = \begin{bmatrix} -525 \\ -567.75 \\ -401.14 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & 0 \\ 0 & 3.55 & 0.9 \\ 0 & 0 & 3.2858 \end{bmatrix}\begin{bmatrix} M_B \\ M_C \\ M_D \end{bmatrix} = \begin{bmatrix} -525 \\ -567.75 \\ -257.2 \end{bmatrix}$$

$M_D = -78.3$

$M_C = -140.1$

$M_B = -96.2$




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